L31: Proving Languages are Not Recognized by any DFA
Which Languages are Recognized by DFAs?

So far we have seen that the following languages have DFAs that recognize them and regular expressions that denote them:
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\left\{ w \in \{a,b\}^* \mid \#_b(w) \equiv 1 \text{ (mod 2)} \right\} \quad L(a^*ba^*(ba^*ba^*)^*)
\]
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So far we have seen that the following languages have DFAs that recognize them and regular expressions that denote them:

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\begin{align*}
\{ w \in \{a, b\}^* \mid \#_b(w) \equiv 1 \pmod{2} \} & \quad \mathcal{L}(a^*ba^*(ba^*ba^*)^*) \\
\{ w \in \{0, 1\}^* \mid \text{next to last symbol is 1} \} & \quad \mathcal{L}((0|1)^*1(0|1))
\end{align*}
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\{ w \in \{0, 1\}^* \mid \text{next to last symbol is 1} \} & \quad \mathcal{L}((0|1)^*1(0|1)) \\
\{ w \in \{a, b\}^* \mid \#_a(w) \equiv 1 \pmod{2} \land \#_b(w) \equiv 0 \pmod{3} \} & \quad ?
\end{align*}
\]
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\{ w \in \{0, 1\}^* \mid \text{next to last symbol is } 1 \} \quad \mathcal{L}((0|1)^*1(0|1))
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\[
\left\{ w \in \{a, b\}^* \mid \#_a(w) \equiv 1 \pmod{2} \land \#_b(w) \equiv 0 \pmod{3} \right\} \quad ?
\]

\[
\{ w \in \{0, 1\}^* \mid w \text{ has at least two } 0\text{'s} \} \quad \mathcal{L}(1^*01^*0(0|1)^*)
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\left\{ w \in \{a, b\}^* \mid \#a(w) \equiv 1 \pmod{2} \land \#b(w) \equiv 0 \pmod{3} \right\}
\]

\[
\{ w \in \{0, 1\}^* \mid w \text{ has at least two 0's} \} \quad \mathcal{L}(1^*01^*0(0|1)^*)
\]

\[
\{ w \in \{0, 1\}^* \mid w \text{ has 001 or 100} \} \quad \mathcal{L}((0|1)^*(001|100)(0|1)^*)
\]
\[ \{ w \in \{0, 1\}^* \mid w \text{ has at least two 0's} \} = \mathcal{L}(1^*01^*0(0|1)^*) \]
\[ \{ w \in \{0, 1\}^* \mid w \text{ has at least two 0's} \} = \mathcal{L}(1^*01^*0(0|1)^*) \]

Build a DFA \( D_4 \) s.t. \( \mathcal{L}(D_4) = \mathcal{L}(1^*01^*0(0|1)^*) \)
\{ w \in \{0, 1\}^* \mid w \text{ has at least two 0's} \} = \mathcal{L}(1^*01^*0(0|1)^*)

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Build a DFA \( D_4 \) s.t. \( \mathcal{L}(D_4) = \mathcal{L}(1^*01^*0(0|1)^*) \)
Which languages are recognized by DFA or denoted by regular expressions?
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How can we prove a language, \( L \), is not recognized by any DFA?
Which languages are recognized by DFA or denoted by regular expressions?

How can we prove a language, $\mathcal{L}$, is not recognized by any DFA?

Idea: must show that need more than a bounded size memory to remember what we have seen so far, $x$, in order to decide if extensions of $x$ belong to $\mathcal{L}$.
Pumping Lemma for Regular Sets:
Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA.
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Let $w \in \mathcal{L}(D)$ s.t. $|w| \geq n$. 


Pumping Lemma for Regular Sets:
Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA.

Let $n = |Q|$.

Let $w \in \mathcal{L}(D)$ s.t. $|w| \geq n$.

Then $\exists x, y, z \in \Sigma^*$ s.t. the following all hold:

1. $xyz = w$
2. $|xy| \leq n$
3. $|y| > 0$, and
4. $\forall k \geq 0 \ (xy^kz \in \mathcal{L}(D))$
Pumping Lemma for Regular Sets:

Let \( D = (Q, \Sigma, \delta, q_0, F) \) be a DFA, \( n = |Q| \)

\( w \in \mathcal{L}(D) \) s.t. \( |w| \geq n \)

Then \( \exists x, y, z \in \Sigma^* \) s.t.

1. \( xyz = w \)
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Then $\exists x, y, z \in \Sigma^*$ s.t. \n
1. $xyz = w$ \n2. $|xy| \leq n$ \n3. $|y| > 0$, and \n4. $\forall k \geq 0 \ (xy^kz \in \mathcal{L}(D))$
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1. $xyz = w$
2. $|xy| \leq n$
3. $|y| > 0$, and
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$n = 3$, $w = 10101 \in \mathcal{L}(D_4)$, $|w| \geq 3$
Pumping Lemma for Regular Sets:

Let \( D = (Q, \Sigma, \delta, q_0, F) \) be a DFA, \( n = |Q| \)

\( w \in \mathcal{L}(D) \) s.t. \( |w| \geq n \)

Then \( \exists x, y, z \in \Sigma^* \) s.t.

1. \( xyz = w \)
2. \( |xy| \leq n \)
3. \( |y| > 0 \), and
4. \( \forall k \geq 0 \) (\( xy^kz \in \mathcal{L}(D) \))

\( n = 3, \ w = 10101 \in \mathcal{L}(D_4), \ |w| \geq 3 \)

Split \( w \) into \( x \cdot y \cdot z \) s.t. \( 1 \land 2 \land 3 \land 4 \)
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Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA, $n = |Q|$

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$x \overset{\text{def}}{=} \epsilon \quad y \overset{\text{def}}{=} 1 \quad z \overset{\text{def}}{=} 0101$
Pumping Lemma for Regular Sets:
Let \( D = (Q, \Sigma, \delta, q_0, F) \) be a DFA,
\[ n = |Q| \]
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Then \( \exists x, y, z \in \Sigma^* \) s.t.
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$n = 3, \ w = 10101 \in \mathcal{L}(D_4), \ |w| \geq 3$

Split $w$ into $x \cdot y \cdot z$ s.t. 1 $\land$ 2 $\land$ 3 $\land$ 4

$x \overset{\text{def}}{=} \epsilon \quad y \overset{\text{def}}{=} 1 \quad z \overset{\text{def}}{=} 0101 \quad \forall k \geq 0 \ (xy^kz \in \mathcal{L}(D_4))$

$k = 0: \ 0101 \in \mathcal{L}(D_4)$,
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1. $xyz = w$
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$n = 3$, $w = 10101 \in \mathcal{L}(D_4)$, $|w| \geq 3$

Split $w$ into $x \cdot y \cdot z$ s.t. 1 $\land$ 2 $\land$ 3 $\land$ 4

$x \overset{\text{def}}{=} \emptyset$ $y \overset{\text{def}}{=} 1$ $z \overset{\text{def}}{=} 0101$ $\forall k \geq 0$ ($xy^kz \in \mathcal{L}(D_4)$)

$k = 0: 0101 \in \mathcal{L}(D_4)$, $k = 1: 10101 \in \mathcal{L}(D_4)$,
Pumping Lemma for Regular Sets:
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$k = 0$ : $0101 \in \mathcal{L}(D_4)$, $k = 1$ : $10101 \in \mathcal{L}(D_4)$, $k = 2$ : $110101 \in \mathcal{L}(D_4)$,
Pumping Lemma for Regular Sets:
Let \( D = (Q, \Sigma, \delta, q_0, F) \) be a DFA, \( n = \| Q \| \)
\( w \in \mathcal{L}(D) \) s.t. \( \| w \| \geq n \)
Then \( \exists x, y, z \in \Sigma^* \) s.t.
\[ 1. \quad xyz = w \\
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3. \quad |y| > 0, \text{ and} \\
4. \quad \forall k \geq 0 \ (xy^k z \in \mathcal{L}(D)) \]

\[ n = 3, \quad w = 10101 \in \mathcal{L}(D_4), \quad \| w \| \geq 3 \]

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\[ k = 0 : \ 0101 \in \mathcal{L}(D_4), \quad k = 1 : \ 10101 \in \mathcal{L}(D_4), \]
\[ k = 2 : \ 110101 \in \mathcal{L}(D_4), \quad k = 3 : \ 1110101 \in \mathcal{L}(D_4), \ldots \]
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3. $|y| > 0$, and
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$n = 3, \quad w = 000 \in \mathcal{L}(D_4), \quad |w| \geq 3$
Pumping Lemma for Regular Sets:
Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA, $n = |Q|$, $w \in \mathcal{L}(D)$ s.t. $|w| \geq n$.
Then, $\exists x, y, z \in \Sigma^*$ s.t. $1 \wedge 2 \wedge 3 \wedge 4$.

$n = 3$, $w = 000 \in \mathcal{L}(D_4)$, $|w| \geq 3$.

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\[ D_4 \]

\[ q_f \]

\( n = 3, \ w = 000 \in \mathcal{L}(D_4), \ |w| \geq 3 \)

**Split** \( w \) into \( x \cdot y \cdot z \) s.t. \( 1 \land 2 \land 3 \land 4 \)

\( x \overset{\text{def}}{=} 00 \quad y \overset{\text{def}}{=} 0 \quad z \overset{\text{def}}{=} \epsilon \)
Pumping Lemma for Regular Sets:

Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA, $n = |Q|$,

$w \in \mathcal{L}(D)$ s.t. $|w| \geq n$,

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$n = 3$, $w = 000 \in \mathcal{L}(D_4)$, $|w| \geq 3$

Split $w$ into $x \cdot y \cdot z$ s.t. $1 \land 2 \land 3 \land 4$

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$k = 0$: $00 \in \mathcal{L}(D_4)$,
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\( w \in \mathcal{L}(D) \) s.t. \( |w| \geq n \)
Then \( \exists x, y, z \in \Sigma^* \) s.t.
\[ \begin{align*}
1. & \quad xyz = w \quad \text{1. } \quad |xy| \leq n \\
2. & \quad |y| > 0, \text{ and} \\
3. & \quad \forall k \geq 0 \ (xy^kz \in \mathcal{L}(D)) \\
\end{align*} \]

\( n = 3, \quad w = 000 \in \mathcal{L}(D_4), \quad |w| \geq 3 \)

Split \( w \) into \( x \cdot y \cdot z \) s.t. 1 \( \land \) 2 \( \land \) 3 \( \land \) 4
\( x \overset{\text{def}}{=} 00, \quad y \overset{\text{def}}{=} 0, \quad z \overset{\text{def}}{=} \epsilon \quad \forall k \geq 0 \ (xy^kz \in \mathcal{L}(D_4)) \)
\( k = 0 : 00 \in \mathcal{L}(D_4), \quad k = 1 : 000 \in \mathcal{L}(D_4), \)
Pumping Lemma for Regular Sets:
Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA, $n = |Q|$ $w \in \mathcal{L}(D)$ s.t. $|w| \geq n$
Then $\exists x, y, z \in \Sigma^*$ s.t. $1 \land 2 \land 3 \land 4$
\[
\begin{align*}
1. & \quad xyz = w \\
2. & \quad |xy| \leq n \\
3. & \quad |y| > 0, \text{ and} \\n4. & \quad \forall k \geq 0 \ (xy^kz \in \mathcal{L}(D))
\end{align*}
\]

$n = 3, \ w = 000 \in \mathcal{L}(D_4), \ |w| \geq 3$

Split $w$ into $x \cdot y \cdot z$ s.t. $1 \land 2 \land 3 \land 4$
\[
\begin{align*}
x \overset{\text{def}}{=} & \ 00 \\
y \overset{\text{def}}{=} & \ 0 \\
z \overset{\text{def}}{=} & \ \epsilon \\
\forall k \geq 0 & \ (xy^kz \in \mathcal{L}(D_4))
\end{align*}
\]
k = 0 : 00 $\in \mathcal{L}(D_4), \ k = 1 : 000 \in \mathcal{L}(D_4), \ k = 2 : 0000 \in \mathcal{L}(D_4)$,
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$n = 3, \ w = 000 \in \mathcal{L}(D_4), \ |w| \geq 3$

Split $w$ into $x \cdot y \cdot z$ s.t. $1 \wedge 2 \wedge 3 \wedge 4$

$x \overset{\text{def}}{=} 00 \quad y \overset{\text{def}}{=} 0 \quad z \overset{\text{def}}{=} \epsilon \quad \forall k \geq 0$ ($xy^kz \in \mathcal{L}(D_4)$)

$k = 0 : 00 \in \mathcal{L}(D_4), \quad k = 1 : 000 \in \mathcal{L}(D_4),$

$k = 2 : 0000 \in \mathcal{L}(D_4), \quad k = 3 : 00000 \in \mathcal{L}(D_4), \ldots$
proof: Let $w \in \mathcal{L}(D)$, $|w| \geq n$, $w = w_1, w_2, \ldots, w_n \cdot u$
proof: Let $w \in \mathcal{L}(D)$, $|w| \geq n$, $w = w_1, w_2, \ldots, w_n \cdot u$

$$w = w_1 \quad w_2 \quad w_3 \quad \cdots \quad w_n \quad u$$

$$q_0 \quad q_1 \quad q_2 \quad q_3 \quad \cdots \quad q_{n-1} \quad q_n \quad q_f$$
proof: Let \( w \in \mathcal{L}(D), \ |w| \geq n, \ w = w_1, w_2, \ldots, w_n \cdot u \)

\[
\begin{array}{cccccccc}
  w &=& w_1 & w_2 & w_3 & \cdots & w_n & u \\
  q_0 & q_1 & q_2 & q_3 & \cdots & q_{n-1} & q_n & q_f \\
\end{array}
\]

By Pigeon-Hole Principle \( \exists i < j \leq n \ (q_i = q_j) \)
proof: Let $w \in \mathcal{L}(D)$, $|w| \geq n$, $w = w_1, w_2, \ldots, w_n \cdot u$

$$w = \underbrace{w_1 \ldots w_i}_{q_0}, \underbrace{w_{i+1} \ldots w_j}_{q_i}, \underbrace{w_{j+1} \ldots w_n u}_{q_i}, w_{n+1} \ldots w_f$$

By Pigeon-Hole Principle $\exists i < j \leq n$ $(q_i = q_j)$

$$w = \underbrace{x}_{w_1 \ldots w_i}, \underbrace{y}_{w_{i+1} \ldots w_j}, \underbrace{z}_{w_{j+1} \ldots w_n u}$$
**proof:** Let \( w \in \mathcal{L}(D) \), \( |w| \geq n \), \( w = w_1, w_2, \ldots, w_n \cdot u \)

\[
w = \begin{array}{cccccccc}
q_0 & q_1 & q_2 & q_3 & \cdots & q_{n-1} & q_n & q_f \\
\end{array}
\]

\[
w = \begin{array}{cccccccc}
w_1 & \ldots & w_i & \ldots & w_{j-1} & w_j & \ldots & w_n \cdot u \\
q_0 & q_i & q_i & \ldots & q_i & q_{j-1} & q_f \\
\end{array}
\]

By **Pigeon-Hole Principle** \( \exists i < j \leq n \) \( (q_i = q_j) \)

\[
q_i = \delta^*(q_i, y) \quad |y| \geq 1 \quad \delta^*(q_i, z) = q_f \in F
\]
**proof:** Let \( w \in \mathcal{L}(D), \ |w| \geq n, \ w = w_1, w_2, \ldots, w_n \cdot u \)

\[
w = \begin{array}{cccccccc}
  w_1 & w_2 & w_3 & \cdots & w_n & u \\
  q_0 & q_1 & q_2 & q_3 & \cdots & q_{n-1} & q_n & q_f
\end{array}
\]

By **Pigeon-Hole Principle** \( \exists i < j \leq n \ (q_i = q_j) \)

\[
w = \begin{array}{cccccccc}
  x & \overbrace{w_1 \ldots w_i}^q & y & \overbrace{w_{i+1} \ldots w_j}^q & z & \overbrace{w_{j+1} \ldots w_n u}^q \\
  q_0 & q_i & q_i & q_i & q_f
\end{array}
\]

\( q_i = \delta^*(q_i, y) \quad |y| \geq 1 \quad \delta^*(q_i, z) = q_f \in F \)

Thus, \( xy^k z \in \mathcal{L}(D) \quad \text{for} \quad k = 0, 1, 2, \ldots \) \( \Box \)
Pumping Lemma for Regular Sets:
Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA.

Let $n = |Q|$.

Let $w \in \mathcal{L}(D)$ s.t. $|w| \geq n$.

Then $\exists x, y, z \in \Sigma^*$ s.t. the following all hold:

1. $xyz = w$
2. $|xy| \leq n$
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4. $\forall k \geq 0 \ (xy^kz \in \mathcal{L}(D))$
Pumping Lemma for Regular Sets:
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Then $\exists x, y, z \in \Sigma^*$ s.t. the following all hold:
1. $xyz = w$
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Easiest tool to prove languages not DFA acceptable
Prop: \( E = \{a^r b^r \mid r \in \mathbb{N}\} \) is not DFA acceptable.

Proof by contradiction:

Assume: \( E \) is accepted by DFA \( D \) with \( n \) states.

you (G) choose string: \( w \in E = L(D) \)

Let \( w = a^n b^n \)

By pumping lemma, \( D \) chooses \( x, y, z \in \{a, b\}^* \), s.t.,

1. \( w = xyz \)
2. \(|xy| \leq n\)
3. \(|y| > 0\), and
4. \( \forall k \in \mathbb{N} \) \((xy^k z) \in E)\)

Since \( 0 < |xy| \leq n \), \( y = a^i \), \( 0 < i \leq n \)

Thus \( xy^0 z = a^{n-i} b^n \in E \).

but \( a^{n-i} b^n \not\in E \).

Therefore \( E \) is not DFA acceptable. \( \square \)
Prop: \( E = \{ a^r b^r \mid r \in \mathbb{N} \} \) is not DFA acceptable.

proof by contradiction:
Assume: \( E \) is accepted by DFA \( D \) with \( n \) states.
Prop: \( E = \{ a^r b^r \mid r \in \mathbb{N} \} \) is not DFA acceptable.

proof by contradiction:
Assume: \( E \) is accepted by DFA \( D \) with \( n \) states.

you (G) choose string: \( w \in E = \mathcal{L}(D) \)
Prop: $E = \{ a^r b^r \mid r \in \mathbb{N} \}$ is not DFA acceptable.

proof by contradiction:
Assume: $E$ is accepted by DFA $D$ with $n$ states.

you (G) choose string: $w \in E = \mathcal{L}(D)$
Let $w = a^n b^n$
Prop: \( E = \{ a^r b^r \mid r \in \mathbb{N} \} \) is not DFA acceptable.

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you (G) choose string: \( w \in E = \mathcal{L}(D) \)
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By pumping lemma, \( D \) chooses \( x, y, z \in \{ a, b \}^* \), s.t.,
1. \( w = a^n b^n = xyz \)
2. \( |xy| \leq n \)
3. \( |y| > 0 \), and
4. \( \forall k \in \mathbb{N} \ ( xy^k z \in E ) \)
Prop: $E = \{ a^r b^r \mid r \in \mathbb{N} \}$ is not DFA acceptable.

proof by contradiction:
Assume: $E$ is accepted by DFA $D$ with $n$ states.

you (G) choose string: $w \in E = \mathcal{L}(D)$
Let $w = a^n b^n$

By pumping lemma, $D$ chooses $x, y, z \in \{ a, b \}^*$, s.t.,

1. $w = a^n b^n = xyz$
2. $|xy| \leq n$
3. $|y| > 0$, and
4. $\forall k \in \mathbb{N} \ ( xy^k z \in E )$

Since $0 < |xy| \leq n$, $y = a^i, 0 < i \leq n$
Prop: $E = \{ a^r b^r \mid r \in \mathbb{N} \}$ is not DFA acceptable.

proof by contradiction:
Assume: $E$ is accepted by DFA $D$ with $n$ states.

you (G) choose string: $w \in E = \mathcal{L}(D)$
Let $w = a^n b^n$

By pumping lemma, $D$ chooses $x, y, z \in \{a, b\}^\star$, s.t.,

1. $w = a^n b^n = xyz$
2. $|xy| \leq n$
3. $|y| > 0$, and
4. $\forall k \in \mathbb{N} \ (xy^k z \in E )$

Since $0 < |xy| \leq n$, $y = a^i$, $0 < i \leq n$

Thus $xy^0 z = a^{n-i} b^n \in E$. 
Prop: $E = \{ a^r b^r \mid r \in \mathbb{N} \}$ is not DFA acceptable.

proof by contradiction:
Assume: $E$ is accepted by DFA $D$ with $n$ states.

you (G) choose string: $w \in E = \mathcal{L}(D)$
Let $w = a^n b^n$

By pumping lemma, $D$ chooses $x, y, z \in \{a, b\}^*$, s.t.,
1. $w = a^n b^n = xyz$
2. $|xy| \leq n$
3. $|y| > 0$, and
4. $\forall k \in \mathbb{N} \ (xy^kz \in E)$

Since $0 < |xy| \leq n$, $y = a^i$, $0 < i \leq n$
Thus $xy^0 z = a^{n-i} b^n \in E$.

but $a^{n-i} b^n \not\in E$. 
Prop: $E = \{ a^r b^r \mid r \in \mathbb{N} \}$ is not DFA acceptable.

**proof by contradiction:**
**Assume:** $E$ is accepted by DFA $D$ with $n$ states.

**you (G) choose string:** $w \in E = \mathcal{L}(D)$
Let $w = a^n b^n$

By **pumping lemma**, $D$ chooses $x, y, z \in \{a, b\}^*$, s.t.,

1. $w = a^n b^n = xyz$
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Since $0 < |xy| \leq n$, $y = a^i$, $0 < i \leq n$

Thus $xy^0 z = a^{n-i} b^n \in E$.

but $a^{n-i} b^n \not\in E$.

F
Prop: \( E = \{ a^r b^r \mid r \in \mathbb{N} \} \) is not DFA acceptable.

proof by contradiction:
Assume: \( E \) is accepted by DFA \( D \) with \( n \) states.

you (G) choose string: \( w \in E = \mathcal{L}(D) \)
Let \( w = a^n b^n \)

By pumping lemma, \( D \) chooses \( x, y, z \in \{a, b\}^* \), s.t.,
1. \( w = a^n b^n = xyz \)
2. \(|xy| \leq n\)
3. \(|y| > 0\), and
4. \( \forall k \in \mathbb{N} \ (xy^k z \in E) \)

Since \( 0 < |xy| \leq n \), \( y = a^i \), \( 0 < i \leq n \)

Thus \( xy^0 z = a^{n-i} b^n \in E \).

but \( a^{n-i} b^n \not\in E \).

\( \square \)

Therefore \( E \) is not DFA acceptable.
Prop: $M = \{ w \in \{a, b\}^* \mid \#_a(w) > \#_b(w) \}$ not DFA-acceptable.

Proof by contradiction:
Assume: $M$ is accepted by DFA $D$ with $n$ states.

You choose string: $w \in M = L(D)$

Let $w = a^n + 1 b^n$.

By pumping lemma, $D$ chooses $x, y, z \in \{a, b\}^*$ s.t.

1. $w = a^n + 1 b^n = xyz$
2. $|xy| \leq n$
3. $|y| > 0$, and
4. $\forall k \in \mathbb{N} (xy^k z \in M)$

Since $0 < |xy| \leq n$, $y = a^i$, $0 < i \leq n$.

Thus $xy^0 z = a^{n + 1 - i} b^n \in M$.

But $a^{n + 1 - i} b^n \not\in M$.

Therefore $M$ is not DFA-acceptable. □
Prop: $M = \{ w \in \{a, b\}^* \mid \#_a(w) > \#_b(w) \}$ not DFA-acceptable.

proof by contradiction:
Assume: $M$ is accepted by DFA $D$ with $n$ states.
Prop: \( M = \{ w \in \{a, b\}^* \mid \#_a(w) > \#_b(w) \} \) not DFA-acceptable.

proof by contradiction:
Assume: \( M \) is accepted by DFA \( D \) with \( n \) states.

you choose string: \( w \in M = \mathcal{L}(D) \)
Prop: $M = \{ w \in \{a, b\}^* \mid \#a(w) > \#b(w) \}$ not DFA-acceptable.

proof by contradiction:
Assume: $M$ is accepted by DFA $D$ with $n$ states.

you choose string: $w \in M = \mathcal{L}(D)$ Let $w = a^{n+1}b^n$
Prop: $M = \{ w \in \{ a, b \}^* \mid \#a(w) > \#b(w) \}$ not DFA-acceptable.

proof by contradiction:
Assume: $M$ is accepted by DFA $D$ with $n$ states.

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By pumping lemma, $D$ chooses $x, y, z \in \{a, b\}^*$ s.t.

1. $w = a^{n+1}b^n = xyz$
2. $|xy| \leq n$
3. $|y| > 0$, and
4. $\forall k \in \mathbb{N} \ (xy^kz \in M)$
Prop: \( M = \{ w \in \{ a, b \}^* \mid \#_a(w) > \#_b(w) \} \) not DFA-acceptable.

**proof by contradiction:**

**Assume:** \( M \) is accepted by DFA \( D \) with \( n \) states.

**you choose string:** \( w \in M = \mathcal{L}(D) \) Let \( w = a^{n+1}b^n \)

By **pumping lemma**, \( D \) chooses \( x, y, z \in \{a, b\}^* \) s.t.

1. \( w = a^{n+1}b^n = xyz \)
2. \( |xy| \leq n \)
3. \( |y| > 0 \), and
4. \( \forall k \in \mathbb{N} \ ( xy^kz \in M ) \)

Since \( 0 < |xy| \leq n \), \( y = a^i \), \( 0 < i \leq n \)
Prop: $M = \{ w \in \{a, b\}^* \mid \#_a(w) > \#_b(w) \}$ not DFA-acceptable.

proof by contradiction:
Assume: $M$ is accepted by DFA $D$ with $n$ states.

you choose string: $w \in M = \mathcal{L}(D)$ Let $w = a^{n+1}b^n$

By pumping lemma, $D$ chooses $x, y, z \in \{a, b\}^*$ s.t.

1. $w = a^{n+1}b^n = xyz$
2. $|xy| \leq n$
3. $|y| > 0$, and
4. $\forall k \in \mathbb{N} \ (xy^kz \in M)$

Since $0 < |xy| \leq n$, $y = a^i$, $0 < i \leq n$

Thus $xy^0z = a^{n+1-i}b^n \in M$. 
Prop: $M = \{ w \in \{a, b\}^* \mid \#a(w) > \#b(w) \}$ not DFA-acceptable.

proof by contradiction:
Assume: $M$ is accepted by DFA $D$ with $n$ states.

you choose string: $w \in M = \mathcal{L}(D)$  Let $w = a^{n+1}b^n$

By pumping lemma, $D$ chooses $x, y, z \in \{a, b\}^*$ s.t.

1. $w = a^{n+1}b^n = xyz$
2. $|xy| \leq n$
3. $|y| > 0$, and
4. $\forall k \in \mathbb{N}$ ($xy^kz \in M$)

Since $0 < |xy| \leq n$, $y = a^i$, $0 < i \leq n$

Thus $xy^0z = a^{n+1-i}b^n \in M$.

but $a^{n+1-i}b^n \not\in M$. 
Prop: \( M = \{ w \in \{a, b\}^* \mid \#a(w) > \#b(w) \} \) not DFA-acceptable.

proof by contradiction:
Assume: \( M \) is accepted by DFA \( D \) with \( n \) states.

you choose string: \( w \in M = \mathcal{L}(D) \) Let \( w = a^{n+1}b^n \)

By pumping lemma, \( D \) chooses \( x, y, z \in \{a, b\}^* \) s.t.

1. \( w = a^{n+1}b^n = xyz \)
2. \(|xy| \leq n\)
3. \(|y| > 0\), and
4. \( \forall k \in \mathbb{N} ( xy^kz \in M ) \)

Since \( 0 < |xy| \leq n \), \( y = a^i, \ 0 < i \leq n \)
Thus \( xy^0z = a^{n+1-i}b^n \in M. \)

but \( a^{n+1-i}b^n \notin M. \)

\( \Box \)
Prop: \( M = \{ w \in \{a, b\}^* \mid \#a(w) > \#b(w) \} \) not DFA-acceptable.

proof by contradiction:
Assume: \( M \) is accepted by DFA \( D \) with \( n \) states.

you choose string: \( w \in M = \mathcal{L}(D) \) Let \( w = a^{n+1}b^n \)

By pumping lemma, \( D \) chooses \( x, y, z \in \{a, b\}^* \) s.t.

1. \( w = a^{n+1}b^n = xyz \)
2. \( |xy| \leq n \)
3. \( |y| > 0 \), and
4. \( \forall k \in \mathbb{N} \ ( xy^kz \in M ) \)

Since \( 0 < |xy| \leq n \), \( y = a^i, 0 < i \leq n \)

Thus \( xy^0z = a^{n+1-i}b^n \in M \).

but \( a^{n+1-i}b^n \notin M \).

F

Therefore \( M \) is not DFA acceptable. \( \square \)
**Prop:** $P = \{ w \in \{a, b\}^* \mid |w| \text{ is prime} \}$ is not DFA acceptable.

by DFA $D$ with $n$ states.
Prop: $P = \{ w \in \{a, b\}^* \mid |w| \text{ is prime} \}$ is not DFA acceptable.

proof: Assume: $P$ is accepted by DFA $D$ with $n$ states.
Prop: \( P = \{ w \in \{a, b\}^* \mid |w| \text{ is prime} \} \) is not DFA acceptable.

proof: Assume: \( P \) is accepted by DFA \( D \) with \( n \) states.

you choose string: \( w \in P = \mathcal{L}(D) \) to get contradiction
Prop: $P = \{ w \in \{a, b\}^* \mid |w| \text{ is prime} \}$ is not DFA acceptable.

proof: Assume: $P$ is accepted by DFA $D$ with $n$ states.

you choose string: $w \in P = \mathcal{L}(D)$ to get contradiction

Let $w = a^p$ where $p \geq n$ is prime
**Prop:** \( P = \{ w \in \{a, b\}^* \mid |w| \text{ is prime} \} \) is not DFA acceptable.

**proof:** Assume: \( P \) is accepted by DFA \( D \) with \( n \) states.

**you choose string:** \( w \in P = \mathcal{L}(D) \) to get contradiction

Let \( w = a^p \) where \( p \geq n \) is prime

By **pumping lemma**, \( D \) chooses \( x, y, z \in \{a, b\}^* \) s.t.

1. \( w = a^p = xyz \)
2. \( |xy| \leq n \)
3. \( |y| > 0 \), and
4. \( \forall k \in \mathbb{N} \ ( xy^k z \in P ) \)
Prop: \( P = \{ w \in \{a, b\}^* \mid |w| \text{ is prime} \} \) is not DFA acceptable.

proof: Assume: \( P \) is accepted by DFA \( D \) with \( n \) states.

you choose string: \( w \in P = \mathcal{L}(D) \) to get contradiction

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\( y = a^i, \ 0 < i \leq n \)
Prop: $P = \{ w \in \{a, b\}^* \mid |w| \text{ is prime} \}$ is not DFA acceptable.

proof: Assume: $P$ is accepted by DFA $D$ with $n$ states.

You choose string: $w \in P = \mathcal{L}(D)$ to get contradiction

Let $w = a^p$ where $p \geq n$ is prime.

By pumping lemma, $D$ chooses $x, y, z \in \{a, b\}^*$ s.t.

1. $w = a^p = xyz$
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3. $|y| > 0$, and
4. $\forall k \in \mathbb{N} \ (xy^k z \in P)$

$y = a^i, \ 0 < i \leq n$

Thus $xy^{p+1}z = xyzy^p = a^p a^{p \cdot i} = a^{p(i+1)} \in P$. 
Prop: \( P = \{ w \in \{a, b\}^* \mid |w| \text{ is prime} \} \) is not DFA acceptable.

proof: Assume: \( P \) is accepted by DFA \( D \) with \( n \) states.

you choose string: \( w \in P = \mathcal{L}(D) \) to get contradiction

Let \( w = a^p \) where \( p \geq n \) is prime

By pumping lemma, \( D \) chooses \( x, y, z \in \{a, b\}^* \) s.t.

1. \( w = a^p = xyz \)
2. \(|xy| \leq n\)
3. \(|y| > 0\), and
4. \( \forall k \in \mathbb{N} ( xy^k z \in P ) \)

\( y = a^i, \: 0 < i \leq n \)

Thus \( xy^{p+1}z = xyzy^p = a^p a^{p \cdot i} = a^{p(i+1)} \in P \).

but \( p(i + 1) \) is not prime, so \( xy^{p+1}z \notin P \).
Prop: $P = \{ w \in \{a, b\}^* \mid |w| \text{ is prime} \} \text{ is not DFA acceptable.}$

proof: Assume: $P$ is accepted by DFA $D$ with $n$ states.

you choose string: $w \in P = \mathcal{L}(D)$ to get contradiction

Let $w = a^p$ where $p \geq n$ is prime

By pumping lemma, $D$ chooses $x, y, z \in \{a, b\}^*$ s.t.

1. $w = a^p = xyz$
2. $|xy| \leq n$
3. $|y| > 0$, and
4. $\forall k \in \mathbb{N} \left( xy^k z \in P \right)$

$y = a^i, \ 0 < i \leq n$

Thus $xy^{p+1}z = xyzy^p = a^p a^p i = a^{p(i+1)} \in P$.

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F
Prop: \( P = \{ w \in \{a, b\}^* \mid |w| \text{ is prime} \} \) is not DFA acceptable.

proof: Assume: \( P \) is accepted by DFA \( D \) with \( n \) states.

you choose string: \( w \in P = \mathcal{L}(D) \) to get contradiction

Let \( w = a^p \) where \( p \geq n \) is prime

By pumping lemma, \( D \) chooses \( x, y, z \in \{a, b\}^* \) s.t.

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2. \(|xy| \leq n\)
3. \(|y| > 0\), and
4. \( \forall k \in \mathbb{N} \ (xy^kz \in P) \)

\( y = a^i, \ 0 < i \leq n \)

Thus \( xy^{p+1}z = xyz^p = a^p a^p i = a^{p(i+1)} \in P \).

but \( p(i+1) \) is not prime, so \( xy^{p+1}z \notin P \).

Therefore \( P \) is not regular. \( \square \)
Pumping Lemma for Regular Sets

Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA.

Let $n = |Q|$.

You (G) choose $w \in L(D)$ s.t. $|w| \geq n$.

Then D chooses $x, y, z \in \Sigma^*$ s.t. the following all hold:

1. $xyz = w$
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Pumping Lemma for Regular Sets

Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA.

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Then D chooses $x, y, z \in \Sigma^*$ s.t. the following all hold:

1. $xyz = w$
2. $|xy| \leq n$
3. $|y| > 0$, and
4. $\forall k \geq 0 \ (xy^kz \in \mathcal{L}(D))$

Finally, you point out why a contradiction ensues.
\{ w \in \{0, 1\}^* \mid w \text{ has 001 or 100} \} = \mathcal{L}((0|1)^*(001|100)(0|1)^*)
\[
\{ w \in \{0, 1\}^* \mid w \text{ has 001 or 100} \} = \mathcal{L}((0|1)^*(001|100)(0|1)^*)
\]

Build an **NFA** \( N_5 \) that accepts \( \mathcal{L}((0|1)^*(001|100)(0|1)^*) \)
\[ \{ w \in \{0, 1\}^* \mid w \text{ has 001 or 100} \} = \mathcal{L}((0|1)^*(001|100)(0|1)^*) \]

Build an NFA \( N_5 \) that accepts \( \mathcal{L}((0|1)^*(001|100)(0|1)^*) \)
\{ w \in \{0, 1\}^* \mid w \text{ has } 001 \text{ or } 100 \} = \mathcal{L}((0|1)^*(001|100)(0|1)^*)

Build an NFA $N_5$ that accepts $\mathcal{L}((0|1)^*(001|100)(0|1)^*)$
\[ \{ w \in \{0, 1\}^* \mid w \text{ has 001 or 100} \} = \mathcal{L}((0|1)^*(001|100)(0|1)^*) \]

Build an **NFA** \( N_5 \) that accepts \( \mathcal{L}((0|1)^*(001|100)(0|1)^*) \)
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Build an NFA $N_5$ that accepts $\mathcal{L}((0|1)^*(001|100)(0|1)^*)$
\[ \{ w \in \{0, 1\}^* \mid w \text{ has 001 or 100} \} = \mathcal{L}((0|1)^*(001|100)(0|1)^*) \]

Build an **NFA** \( N_5 \) that accepts \( \mathcal{L}((0|1)^*(001|100)(0|1)^*) \)
Kleene’s Theorem Let \( A \subseteq \Sigma^* \) be any language. Then the following are equivalent:

1. \( A = \mathcal{L}(D) \), for some DFA \( D \).
2. \( A = \mathcal{L}(N) \), for some NFA \( N \) wo \( \epsilon \) transitions.
3. \( A = \mathcal{L}(N) \), for some NFA \( N \).
4. \( A = \mathcal{L}(e) \), for some regular expression \( e \).
5. \( A \) is regular.
Kleene’s Theorem Let $A \subseteq \Sigma^*$ be any language. Then the following are equivalent:

1. $A = \mathcal{L}(D)$, for some DFA $D$.
2. $A = \mathcal{L}(N)$, for some NFA $N$ wo $\epsilon$ transitions.
3. $A = \mathcal{L}(N)$, for some NFA $N$.
4. $A = \mathcal{L}(e)$, for some regular expression $e$.
5. $A$ is regular.

True by definition: $(4) \iff (5)$