CS250: Discrete Math for Computer Science

L31: Proving Languages are Not Recognized by any DFA

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$\left\{ w \in \{a,b\}^\star \;\middle|\; \begin{array}{l} \#_a(w) \equiv 1 \pmod{2} \wedge \\ \#_b(w) \equiv 0 \pmod{3} \end{array} \right\}$	?

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$\left\{ w \in \{0,1\}^* \;\middle|\; w \text{ has at least two 0's} \right\}$	$\mathcal{L}(1^*01^*0(0\|1)^*)$
$\left\{ w \in \{0,1\}^* \;\middle|\; w \text{ has 001 or 100} \right\}$	$\mathcal{L}((0\|1)^*(001\|100)(0\|1)^*)$

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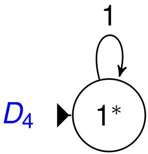

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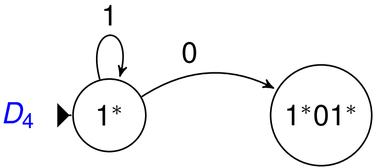

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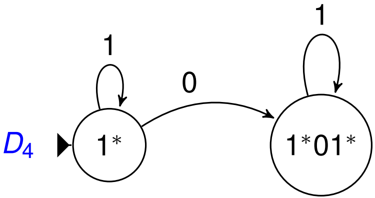

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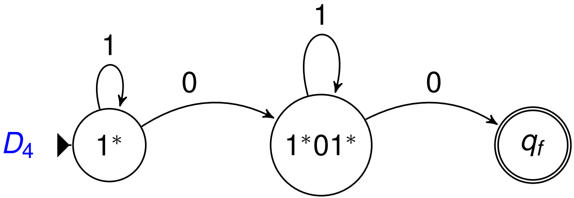

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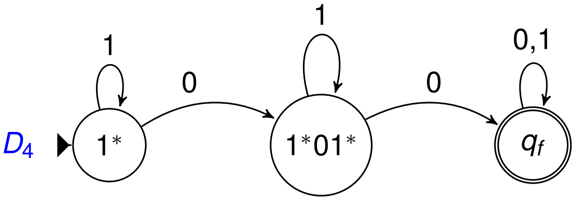

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How can we **prove** a language, $\mathcal{L}$, is **not** recognized by **any DFA**?

Idea: must show that need **more** than a **bounded** size memory to **remember** what we have seen so far, x, in order to decide if **extensions** of x belong to $\mathcal{L}$.

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Then $\quad \exists x, y, z \in \Sigma^\star \quad$ s.t. the following all hold:

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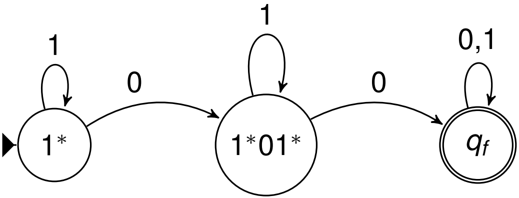

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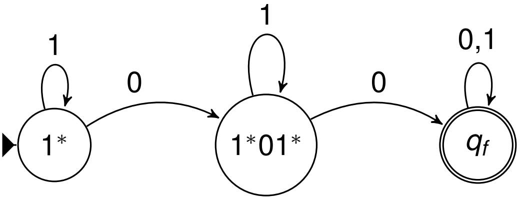

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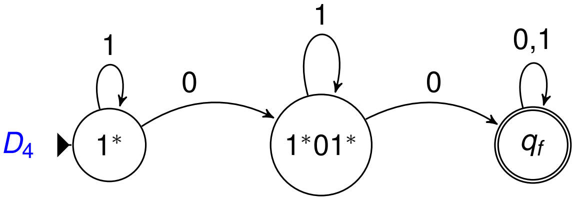

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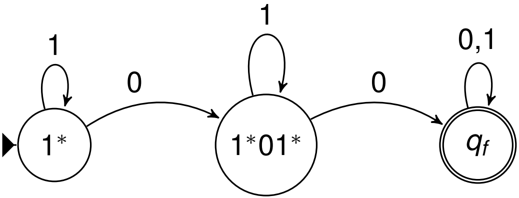

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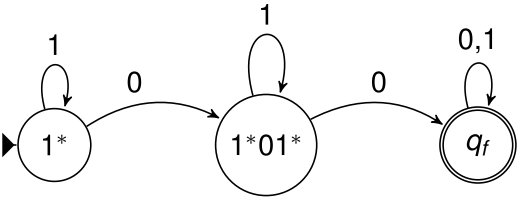

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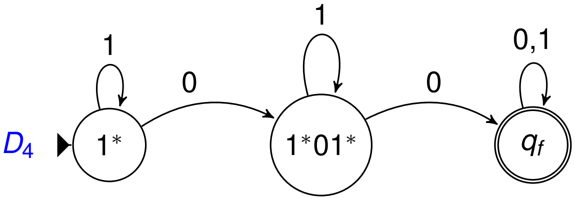

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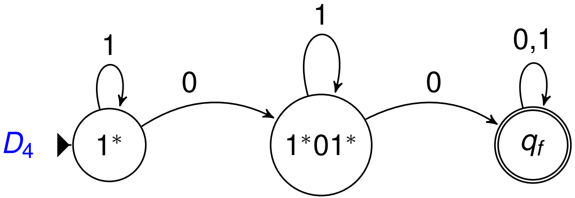

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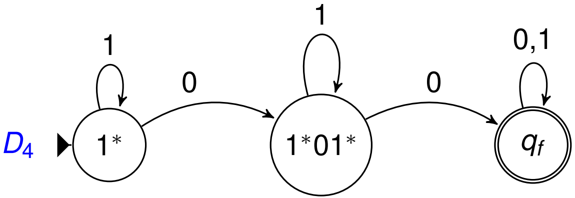

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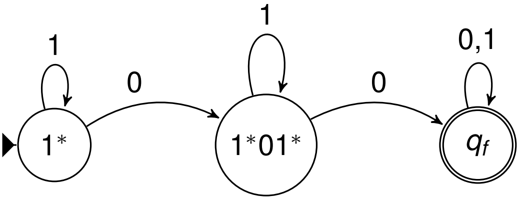

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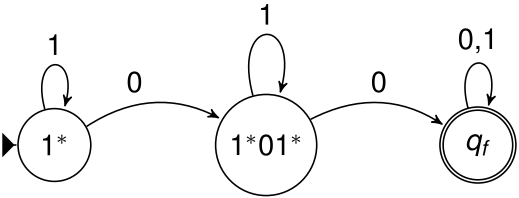

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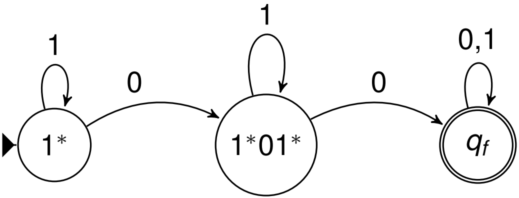

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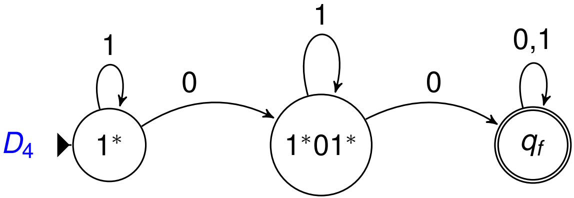

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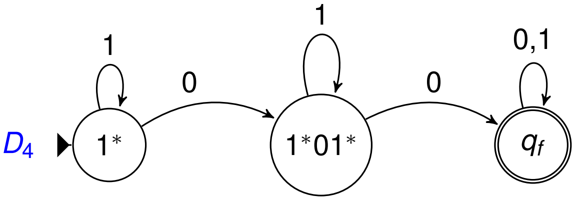

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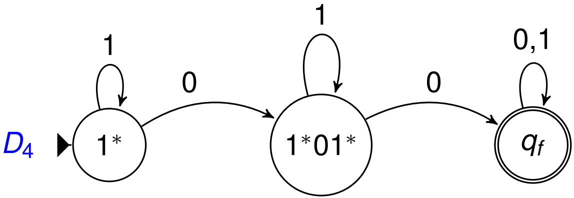

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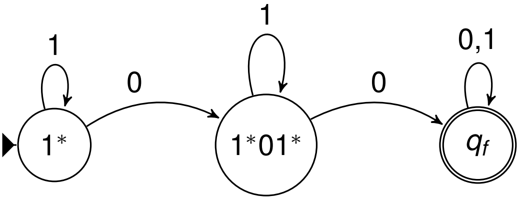

$n = 3, \quad w = 000 \in \mathcal{L}(D_4), \quad |w| \geq 3$

Split $\quad w \quad$ into $\quad x \cdot y \cdot z \quad$ s.t. $\;1 \wedge 2 \wedge 3 \wedge 4$

$x \stackrel{\text{def}}{=} 00 \quad y \stackrel{\text{def}}{=} 0 \quad z \stackrel{\text{def}}{=} \epsilon \qquad \forall k \geq 0 \quad (xy^k z \in \mathcal{L}(D_4))$

$k = 0 : \; 00 \in \mathcal{L}(D_4),$

Pumping Lemma for Regular Sets:

Let $D = (Q, \Sigma, \delta, q_0, F)$

be a DFA, $\quad n = |Q|$

$w \in \mathcal{L}(D)$ s.t. $|w| \geq n$

Then $\quad \exists x, y, z \in \Sigma^\star$ s.t.

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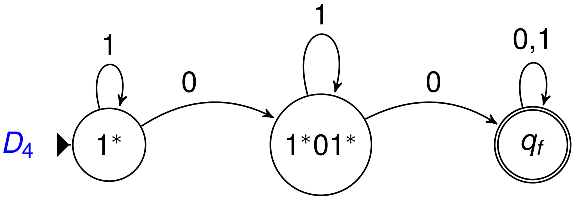

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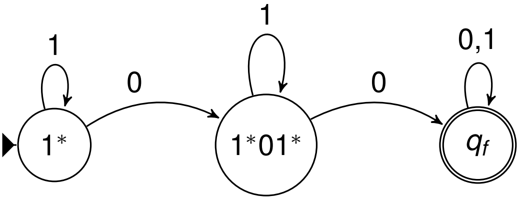

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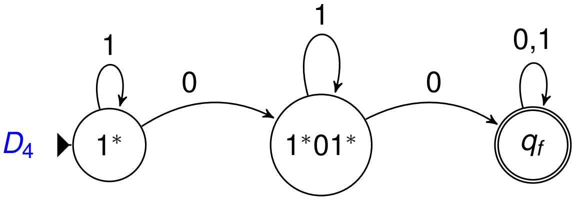

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$k = 2:\; 0000 \in \mathcal{L}(D_4), \quad k = 3:\; 00000 \in \mathcal{L}(D_4), \ldots$

proof: Let $w \in \mathcal{L}(D)$, $\quad |w| \geq n$, $\quad w \;=\; w_1, w_2, \ldots, w_n \cdot u$

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$$\begin{array}{rccccccccccccccc} w = & & w_1 & & w_2 & & w_3 & & \cdots & & & w_n & & u & \\ & q_0 & & q_1 & & q_2 & & q_3 & \cdots & q_{n-1} & & & q_n & & q_f \end{array}$$

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By **Pigeon-Hole Principle** $\quad \exists i < j \leq n \quad (q_i = q_j)$

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By **Pigeon-Hole Principle** $\exists i < j \leq n \quad (q_i = q_j)$

$$w = \quad q_0 \; \overbrace{w_1 \ldots w_i}^{x} \; q_i \; \overbrace{w_{i+1} \ldots w_j}^{y} \; q_i \; \overbrace{w_{j+1} \ldots w_n u}^{z} \; q_f$$

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Thus, $xy^kz \in \mathcal{L}(D)$ $\quad$ for $\quad k = 0, 1, 2, \ldots$ $\qquad \square$

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Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA.

Let $n = |Q|$.

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Easiest tool to prove languages not DFA acceptable

Prop: $E = \left\{ a^r b^r \;\middle|\; r \in \mathbf{N} \right\}$ is not DFA acceptable.

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1. $w = a^n b^n = xyz$
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Since $0 < |xy| \leq n$, $\quad y = a^i,\; 0 < i \leq n$

Thus $xy^0 z = a^{n-i} b^n \in E$.

Prop: $E = \{ a^r b^r \mid r \in \mathbf{N} \}$ is not DFA acceptable.

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but $a^{n-i} b^n \notin E$.

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Therefore E is **not DFA acceptable**. $\square$

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Let $w = a^p$ where $p \geq n$ is prime

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$y = a^i, \; 0 < i \leq n$

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Let $w = a^p$ where $p \geq n$ is prime

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1. $w = a^p = xyz$
2. $|xy| \leq n$
3. $|y| > 0$, and
4. $\forall k \in \mathbf{N}\ (\ xy^k z \in P\ )$

$y = a^i,\ 0 < i \leq n$

Thus $xy^{p+1}z = xyzy^p = a^p a^{p \cdot i} = a^{p(i+1)} \in P$.

but $p(i+1)$ is **not prime**, so $xy^{p+1}z \notin P$.

F

Therefore P is **not regular**. $\square$

Pumping Lemma for Regular Sets

Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA.

Let $n = |Q|$.

You (**G**) choose $w \in \mathcal{L}(D)$ s.t. $|w| \geq n$.

Then **D** chooses $x, y, z \in \Sigma^\star$ s.t. the following all hold:

1. $xyz = w$
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Finally, you point out why a contradiction ensues.

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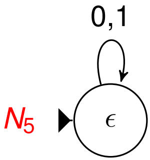

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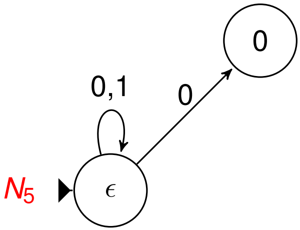

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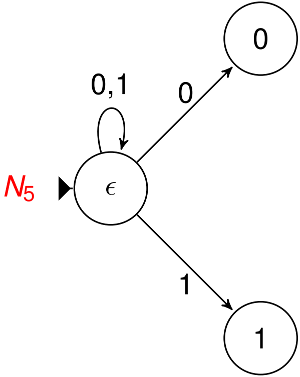

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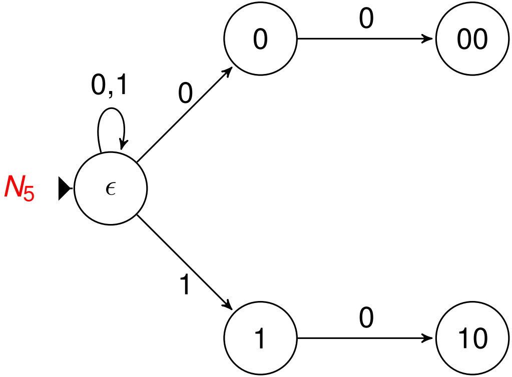

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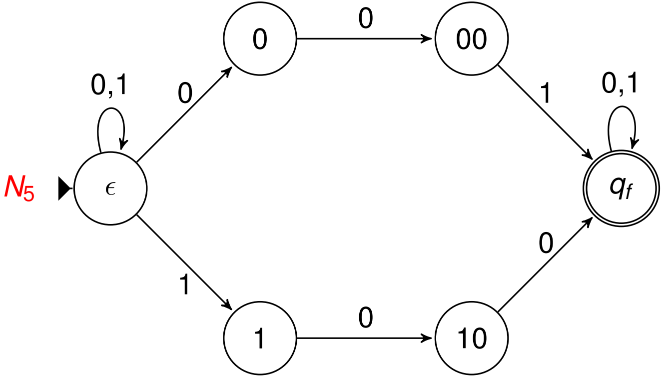

Goal for Friday

Kleene's Theorem Let $A \subseteq \Sigma^\star$ be any language. Then the following are equivalent:

1. $A = \mathcal{L}(D)$, for some DFA D.
2. $A = \mathcal{L}(N)$, for some NFA N wo ϵ transitions.
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True by definition: (4) $\Leftrightarrow$ (5)