CS250: Discrete Math for Computer Science

L1: Course Overview & Intro to Language of Math

What is this course about?

Rigorous introduction to discrete mathematics and language of math.

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Main Goal: become fluent in the language of math.

- **precise specifications**: **learn how to say exactly what you mean**
- **proofs**: **learn how to be absolutely certain that what you say is correct**

Plan and Requirements for Course

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- **Extra Credit:** I will reward useful help to your fellow students, such as reporting typos or errors in the slides, handouts, or quizzes. I will also reward helpful questions or helpful answers on Piazza.

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- Cheating is **harmful**, **counter-productive** and **quite dangerous** to your academic career.

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- Post on Piazza any remaining questions you have on the reading or on sample problems you tried.
- Try to answer other students' posted questions.
- Read the CICS Inclusiveness Statement, posted on the CS250 homepage. You are not competing with your fellow student. You are all trying to learn to understand and use the language of math. I would be **delighted** if everyone in this class passes with a C or better.

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- Statements 6 and 7 are both existential. Statement 6 is true and 7 is false. (In §1.2 we will talk about sets.)

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- One way to show that a universal statement is false is to find a **counterexample**.
- 2 is a counter-example to universal statement 6.

Conditional Statements: if A then B

A **conditional** statement, $A \rightarrow B$ (A implies B), says if the **hypothesis**, A, is true, then so must be the **conclusion** B.

- $A \rightarrow B$ is a contract: I promise if you fulfill A, then I will fulfill B. If you do not fulfill A, then I am under no obligation.

	A	B	not A	A and B	A or B	A implies B	not A or B
W	A	B	$\sim A$	$A \wedge B$	$A \vee B$	$A \rightarrow B$	$\sim A \vee B$
W_3	1	1	0	1	1	1	1
W_2	1	0	0	0	1	0	0
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- $A \to B \equiv \sim A \vee B$, only fails when A is true and B is false.

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Universal Conditional Statements $\forall x(A(x) \to B(x))$

1. Most universal statements contain a conditional part.

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6. For all dogs, a, a is a mammal.
7. All dogs are mammals.

Existential Universal Statements $\exists x \, \forall y \; P(x, y)$

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3. There exists $x \in \mathbf{R}$ such that for all $y \in \mathbf{R}$, $x \cdot y = y$.

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6. For all $x \in R$, if $x \neq 1$ then, there exists $y \in \mathbf{R}$ such that $x \cdot y = 1$.

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