Make sure that you understand the Natural Deduction proof rules for PropCalc and how they are used to justify the following correct proof that $\vdash (\sim p \vee \sim q) \rightarrow \sim(p \wedge q)$. The discussion leader will go over this proof – please ask any questions and make sure that you understand.

## Natural Deduction Rules for PropCalc

| | introduction | elimination |
|---|---|---|
| $\wedge$ | $\dfrac{p \quad q}{p \wedge q}$ | $\dfrac{p \wedge q}{p} \quad \dfrac{p \wedge q}{q}$ |
| $\vee$ | $\dfrac{p}{p \vee q} \quad \dfrac{q}{p \vee q}$ | $\dfrac{p \vee q \quad p \vdash r \quad q \vdash r}{r}$ |
| $\rightarrow$ | $\dfrac{p \vdash q}{p \rightarrow q}$ | $\dfrac{p \rightarrow q \quad p}{q} \quad \dfrac{p \rightarrow q \quad \sim q}{\sim p}$ |
| **F** | $\dfrac{p \quad \sim p}{\mathbf{F}}$ | $\dfrac{p \vdash \mathbf{F}}{\sim p} \quad \dfrac{\sim p \vdash \mathbf{F}}{p}$ |
| $\sim\sim$ | $\dfrac{p}{\sim\sim p}$ | $\dfrac{\sim\sim p}{p}$ |

**Example:** $\vdash (\sim p \vee \sim q) \rightarrow \sim(p \wedge q)$

| | | |
|---|---|---|
| 1 | $\sim p \vee \sim q$ | |
| 2 | $p \wedge q$ | |
| 3 | $\sim p$ | |
| 4 | $p$ | $\wedge$-e, 2 |
| 5 | **F** | **F**-i, 3, 4 |
| 6 | $\sim q$ | |
| 7 | $q$ | $\wedge$-e, 2 |
| 8 | **F** | **F**-i, 6, 7 |
| 9 | **F** | $\vee$-e, 1, 3–5, 6–8 |
| 10 | $\sim(p \wedge q)$ | **F**-e, 2–9 |
| 11 | $(\sim p \vee \sim q) \rightarrow \sim(p \wedge q)$ | $\rightarrow$-i, 1–10 |

Then work together in your group to fill in the correct justifications for the following Natural Deduction proof of $\vdash (p \vee \sim p)$.

As usual, there will be a D3 Quiz on Moodle tonight in which you should use what you learned in the discussion today to fill in answers to some multiple choice questions.

## Natural Deduction Rules for PropCalc

| | introduction | elimination |
|---|---|---|
| $\wedge$ | $\dfrac{p \quad q}{p \wedge q}$ | $\dfrac{p \wedge q}{p} \quad \dfrac{p \wedge q}{q}$ |
| $\vee$ | $\dfrac{p}{p \vee q} \quad \dfrac{q}{p \vee q}$ | $\dfrac{p \vee q \quad p \vdash r \quad q \vdash r}{r}$ |
| $\rightarrow$ | $\dfrac{p \vdash q}{p \rightarrow q}$ | $\dfrac{p \rightarrow q \quad p}{q} \quad \dfrac{p \rightarrow q \quad \sim q}{\sim p}$ |
| **F** | $\dfrac{p \quad \sim p}{\mathbf{F}}$ | $\dfrac{p \vdash \mathbf{F}}{\sim p} \quad \dfrac{\sim p \vdash \mathbf{F}}{p}$ |
| $\sim\sim$ | $\dfrac{p}{\sim\sim p}$ | $\dfrac{\sim\sim p}{p}$ |

$$\vdash \quad \sim(p \vee \sim p)$$

| | | | |
|---|---|---|---|
| 1 | | $\sim (p \vee \sim p)$ | |
| 2 | | $p$ | |
| 3 | | $p \vee \sim p$ | $\square$, 2 |
| 4 | | $\mathbf{F}$ | $\square$, 1, 3 |
| 5 | | $\sim p$ | $\square$, 2–4 |
| 6 | | $p \vee \sim p$ | $\square$, 5 |
| 7 | | $\mathbf{F}$ | $\square$, 1, 6 |
| 8 | $(p \vee \sim p)$ | | $\square$, 1–7 |