Computing Metamers

Erik G. Learned-Miller
Department of Computer Science
University of Massachusetts, Amherst
Amherst, MA 01003

September 17, 2012

Abstract

1 Definitions

In this document we shall describe spectra of light and the response of sensors to light as discrete functions. The continuous spectrum from 380 nanometers to 780 nanometers of light will be divided into 400 sections of 1 nanometer each. When we refer to the power at a given frequency, say 395 nanometers, we are really referring to the total power in the range of continuous frequencies between 395.0 nanometers and 396.0 nanonmeters. This will allow us to use discrete mathematics (sums and dot products) to explain our metamer calculations rather than integrals.

Problem definition. Consider a *target* light that causes a certain response from the human eye by stimulating the cone cells in the eye. Given three flashlights with different spectral distributions of light, our goal will be to combine certain amounts of these lights (by turning a "power" dial on each light) to reproduce the effect of the original target light.

- Let $T(\lambda)$ be the power of the target light at each discrete wavelength λ that is entering the eye in units of nanowatts.
- Let l_1, l_2, and l_3 be the three light sources whose power we can control, and $l_1(\lambda)$, $l_2(\lambda)$, and $l_3(\lambda)$ represent the power (in nanowatts entering the eye) of each of these lights at each discrete frequency.
- Let $S(\lambda)$, $M(\lambda)$, and $L(\lambda)$ represent the percentage of power absorbed at each integer wavelength of light by the *short*, *medium*, and *long* cone cells of the retina. That is, these are the absorption profiles for each type of cone cell.
- Let $\mathbf{R}$ be a vector of three components R_S, R_M, and R_L, that represents the responses of the three types of cone cells to the total target light.
- Finally, the variables A, B, and C represent the power settings that we can set on our three flashlights (A for l_1, B for l_2 and C for l_3) in an attempt to reproduce the response of the target light.

2 Step 1

If we are not given the values of R_S, R_M, and R_L, then the first step should be to compute these values. Each of these will be a simple dot product, taking the amount of light from the target at a given frequency, multiplying this by the absorption of a given cone cell type, and summing these up.

We have:

$$R_S = \sum_{\lambda=380}^{779} S(\lambda)T(\lambda),$$

$$R_M = \sum_{\lambda=380}^{779} M(\lambda)T(\lambda),$$

and

$$R_L = \sum_{\lambda=380}^{779} L(\lambda)T(\lambda).$$

3 Step 2

Now that we have the desired responses from the short, medium, and long cones, we have to find out how to reproduce these responses using the flashlights.

In particular, we wish to find power adjustments A, B, and C such that we get the appropriate response R_S, R_M, and R_L from the cone cells:

$$S \cdot Al_1 + S \cdot Bl_2 + S \cdot Cl_3 = R_S,$$

$$M \cdot Al_1 + M \cdot Bl_2 + M \cdot Cl_3 = R_M$$

and

$$L \cdot Al_1 + L \cdot Bl_2 + L \cdot Cl_3 = R_L$$

The coefficients A, B, and C can be brought outside the dot products, since the dot product is a linear operation to yield:

$$A(S \cdot l_1) + B(S \cdot l_2) + C(S \cdot l_3) = R_S,$$

$$A(M \cdot l_1) + B(M \cdot l_2) + C(M \cdot l_3) = R_M$$

and

$$A(L \cdot l_1) + B(L \cdot l_2) + C(L \cdot l_3) = R_L$$

Next, compute the dot products inside the parentheses and rename them to be elements of a matrix, like this:

$$AD(1,1) + BD(1,2) + CD(1,3) = R_S,$$

$$AD(2,1) + BD(2,2) + CD(2,3) = R_M,$$

and

$$AD(3,1) + BD(3,2) + CD(3,3) = R_L.$$

This can be rewritten as a matrix equation:

$$D\mathbf{v} = \mathbf{R}.$$

Applying the inverse of the matrix D to both sides, we have

$$D^{-1}D\mathbf{v} = D^{-1}\mathbf{R}.$$

and finally

$$\mathbf{v} = D^{-1}\mathbf{R}.$$

In summary, you first need to calculate the cone responses, then the elements of the matrix D, and then the inverse of the matrix D. Finally, you apply the inverse of D to the vector of responses $\mathbf{R}$ to yield the vector of desired coefficients A, B, and C.

4 Comments

We have made a few simplifying assumptions in the above text.

4.1 Color gamuts

The *color gamut* of a set of light sources is the set of colors that can be reproduced using varying combinations of those light sources.

First, the power spectra of the lights l_1, l_2, and l_3 must be *linearly independent* in order to generate a gamut that is three-dimensional. That is, no linear combination of the first two lights should yield the third light source. For example, if the third light source is simply the sum of the other two light sources (i.e., $l_3 = l_1 + l + 2$), then the third light source does not make a contribution to the gamut, and we will find ourselves with a two-dimensional (and hence, highly impoverished) gamut.

5 Non-negative combinations

Another simplification we have made is to assume that there exist *non-negative numbers* A, B, and C, such that the target light can be reproduced. If, upon solving the equations above, the values of either A, B, or C is negative, then this implies that it is impossible to match the target source using the light sources l_1, l_2, and l_3, since it is impossible to produce a "negative" quantity of light from a particular source.