SPIKING NEURAL NETWORKS: AN ALGORITHMIC PERSPECTIVE

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Massachusetts Institute of Technology, EECS.
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BDA 2017.

Based on work in:

- *Computational Tradeoffs in Biological Neural Networks: Self-Stabilizing Winner-Take-All Networks*. ITCS 2017.
- *Neuro-RAM Unit with Applications to Similarity Testing and Compression in Spiking Neural Networks*. DISC 2017.

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Full versions are available at: `cameronmusco.com`

High Level Goal: Understand how computation is performed in biological neural networks at an algorithmic level.

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NEURAL CIRCUITS FROM AN ALGORITHMIC VIEW

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We focus on fixed networks and do not (yet) consider how they are learned. Our tasks are basic computational primitives rather than more complex pattern recognition goals.

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- Significantly influenced by the work of Wolfgang Maass on the theory of spiking neural networks.

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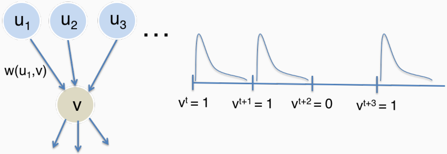

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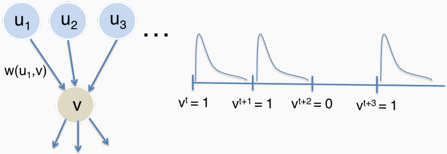

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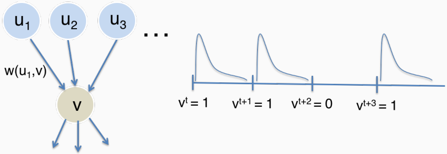

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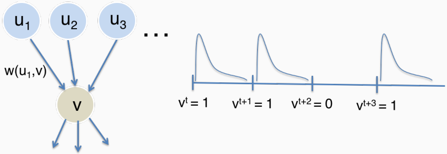

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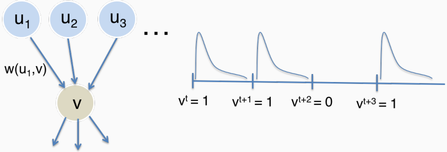

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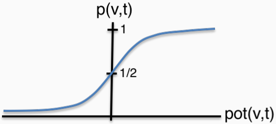
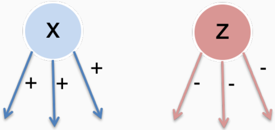


All neurons are strictly inhibitory or excitatory – i.e. $w(u, v) \geq 0$ for all v or $w(u, v) \leq 0$ for all v.



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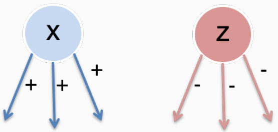

Ignore many other biological features. E.g. refractory period, spike propagation delay, memory, noise on synapses etc. Some can be simulated in our model.

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- n input neurons X each either always firing or not firing.
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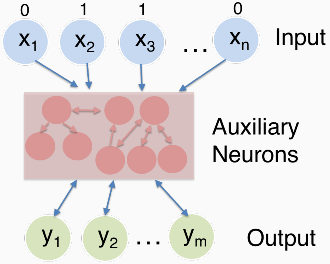
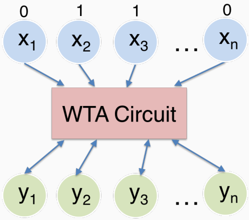


Questions so far?

Example Problem 1

Binary WTA problem: Want to converge to a single firing output, which corresponds to a firing input.



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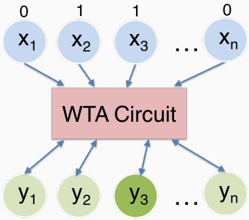

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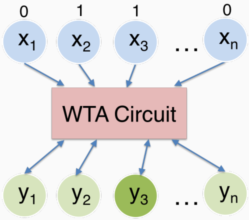

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WINNER-TAKE-ALL (WTA)

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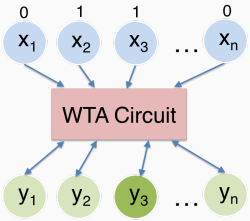
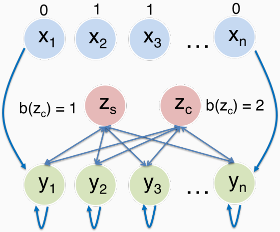
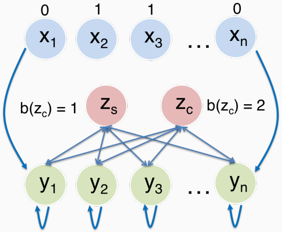


- Neural leader election. Very heavily studied in computational neuroscience.
- Used in perceptual attention, competitive learning, etc. Powerful 'nonlinear' primitive [Maass '99]

SIMPLE SOLUTION WITH TWO INHIBITORS





Main idea: Inhibitors facilitate competition (or lateral inhibition) between inputs, leading to a single 'winner'.

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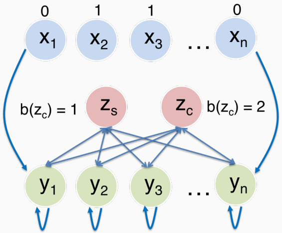
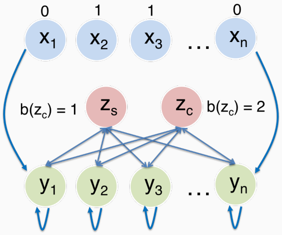


- Stability inhibitor z_s fires whenever there are ≥ 1 competing outputs and prevents any output that didn't fire at time t from firing at time $t + 1$.



- Stability inhibitor z_s fires whenever there are ≥ 1 competing outputs and prevents any output that didn't fire at time t from firing at time $t+1$.
- Convergence inhibitor z_c fires whenever there are ≥ 2 competing outputs and causes any competing output to stop firing at time $t+1$ with probability $1/2$.

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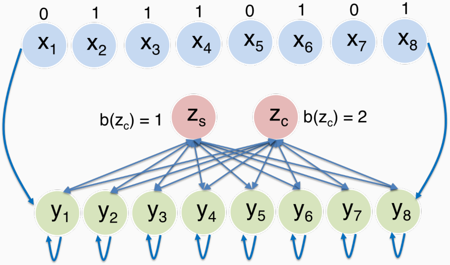

- Roughly 1/2 of competing outputs stop firing at each time step. With constant probability there is some time $t \leq \log n$ such that exactly one output fires at time t.
- After time t, this distinguished output continues to fire. Just z_s fires, preventing all other outputs from firing.

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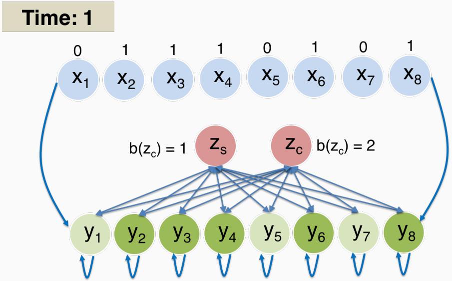

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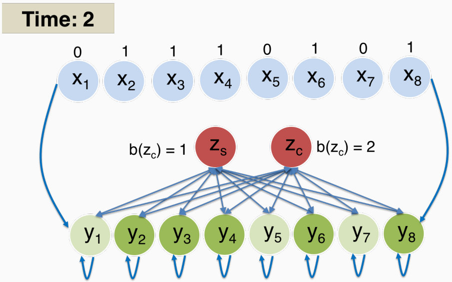

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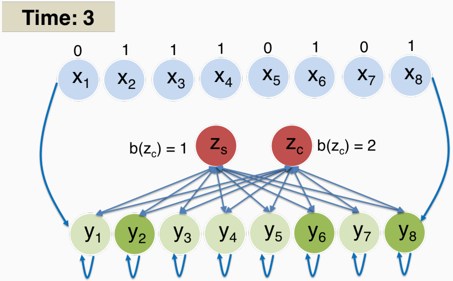

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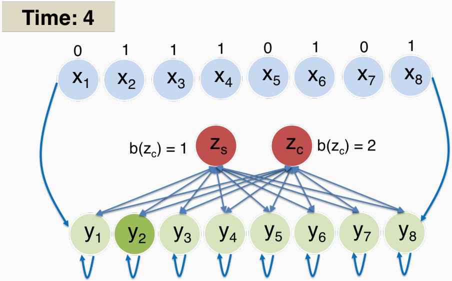

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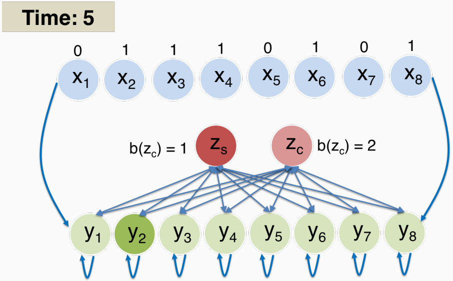

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- More than two inhibitors can be used to give faster convergence (see full paper for results characterizing this tradeoff.)
- Can be used to solve non-binary WTA. Goal here is to select the input with the highest firing rate.

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2. Inhibitors behave nearly deterministically. Randomness is used solely for symmetry breaking between outputs. Highlights dual nature of randomness – can be a powerful computational resource but can also slow down computation by leading to noisy behavior.

Example Problem 2

Similarity Testing: Given two input firing patterns X_1 and X_2, distinguish whether $X_1 = X_2$ or if they are far from being equal. I.e. if $d(X_1, X_2) \geq \epsilon n$.

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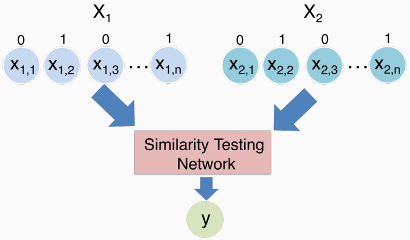

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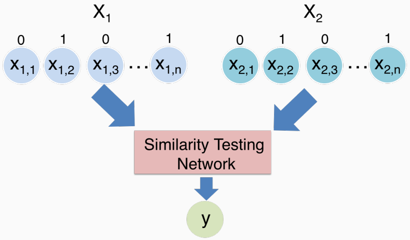

- Natural sub-problem for pattern recognition and other tasks.

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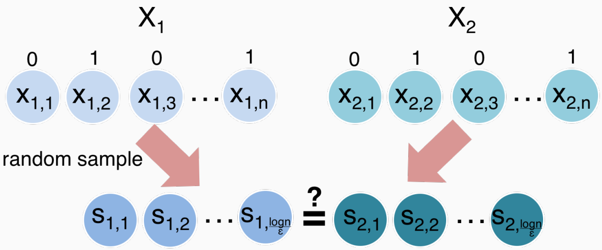

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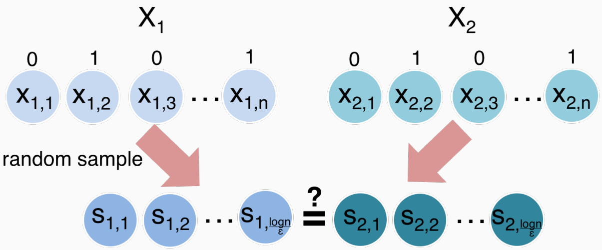

If $X_1 = X_2$, then $S_1 = S_2$. If $d(X_1, X_2) \geq \epsilon n$, the $S_1 \neq S_2$ with high probability.

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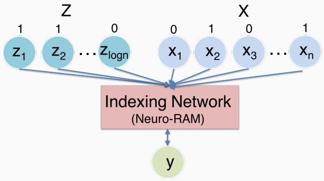

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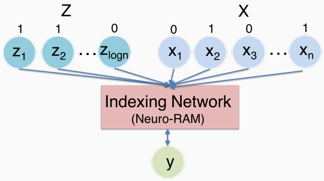

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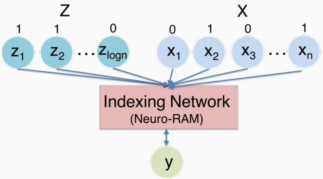

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- Simulates an excitatory connection from $X(Z)$ to y.

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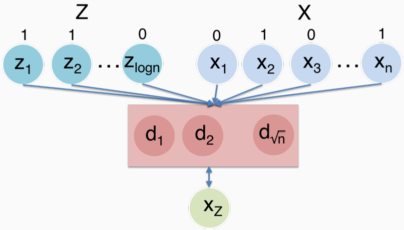
- This seems to be an important primitive in many computations beyond our similarity testing application. E.g. a smell or sight triggering a memory.

Theorem

For any $t \leq \sqrt{n}$, there is an SNN solving the indexing problem with $O(n/t)$ auxiliary neurons that converges in t time steps with high probability. For $t = \sqrt{n}$, the circuit uses $O(\sqrt{n})$ auxiliary neurons.



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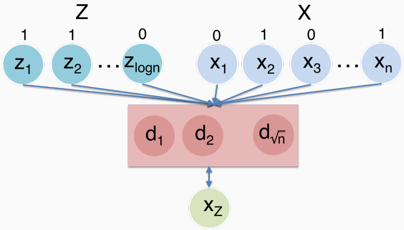

- Gives an $O\left(\frac{\sqrt{n}\log n}{\epsilon}\right)$ (i.e., sublinear) sized circuit for the similarity testing problem.

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- The same upper bound can be achieved with linear threshold gates (i.e. perceptrons)
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- Also separates our model from sigmoidal gates with real valued outputs which can implement indexing with $O(\sqrt{n})$ neurons converging in $O(1)$ steps.

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  - Any compact indexing network must converge slowly, and thus seems somewhat unlikely as a neural implementation.
- Is general indexing machinery actually implemented in the brain?
- Our similarity testing algorithm is a simple application of randomized compression.

- Our result can be seen in two ways:
  - Indexing can be implemented with a compact spiking networks.
  - Any compact indexing network must converge slowly, and thus seems somewhat unlikely as a neural implementation.
- Is general indexing machinery actually implemented in the brain?
- Our similarity testing algorithm is a simple application of randomized compression.
  - Other randomized compression schemes like Johnson-Lindenstrauss projection have been considered as possible neural algorithms.
  - To what extent are these schemes implemented via random connectivity and to what extent do they require indexing operations?

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- Theoretical abstractions that let us handle biological complexity.
- What features of our model can be generalized? E.g. can we prove results for a wider class of activation functions beyond the sigmoid?

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- How do fixed network motifs such as WTA and similarity testing circuits interact with more flexible 'learning' networks?
- More generally, would like to develop a theory for composing spiking neural networks to solve complex problems.

Thanks!