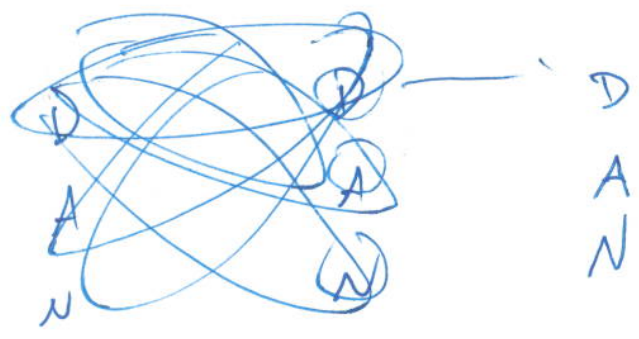


Forward Alg

Want:  $P(w) = \sum_y P(w, y)$

$$\sum_y \prod_t P(w_t | y_t) P(y_t | y_{t-1})$$

y      y      y  
 the            happy            dog



the            happy            dog

N-th order HMM ?  
 $P(y_t | y_{t-1} y_{t-2})$

1st order F. Algo.

Runtime ?

$K$  state vocab.

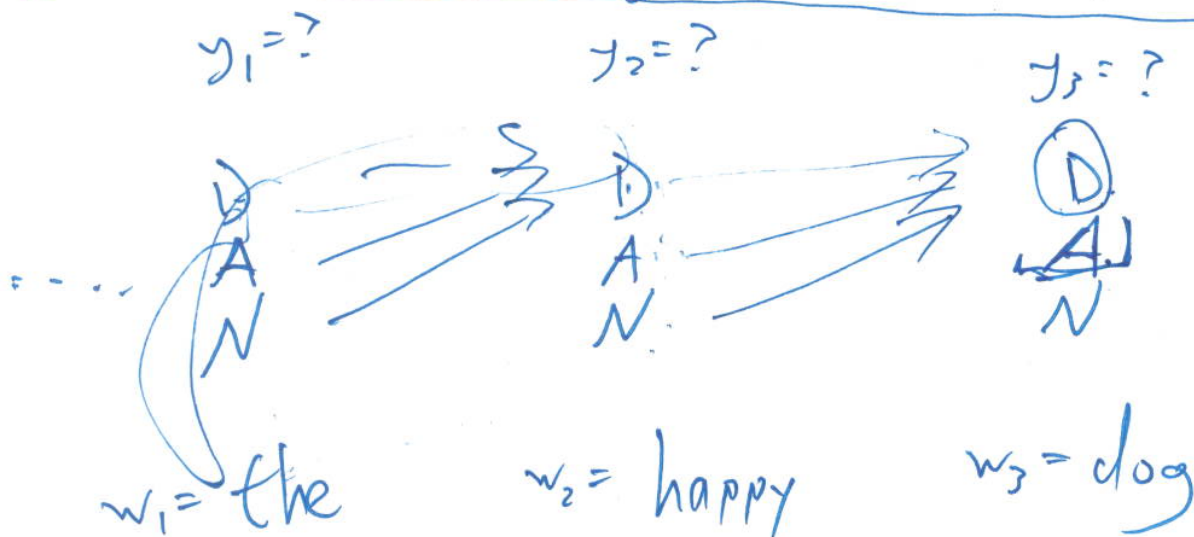
$N$  tokens long

$K^N$ : Naive Enum Alg.

$O(KN)$ : F. Algo

Want  $P(\vec{w}) = \sum_{\vec{y}} P(\vec{w}, \vec{y})$

$$= \sum_{\vec{y}} \prod_t P(y_t | y_{t-1}) P(w_t | y_t)$$



Forward Algo = left-to-right

Log-linear form for an HMM — for handout

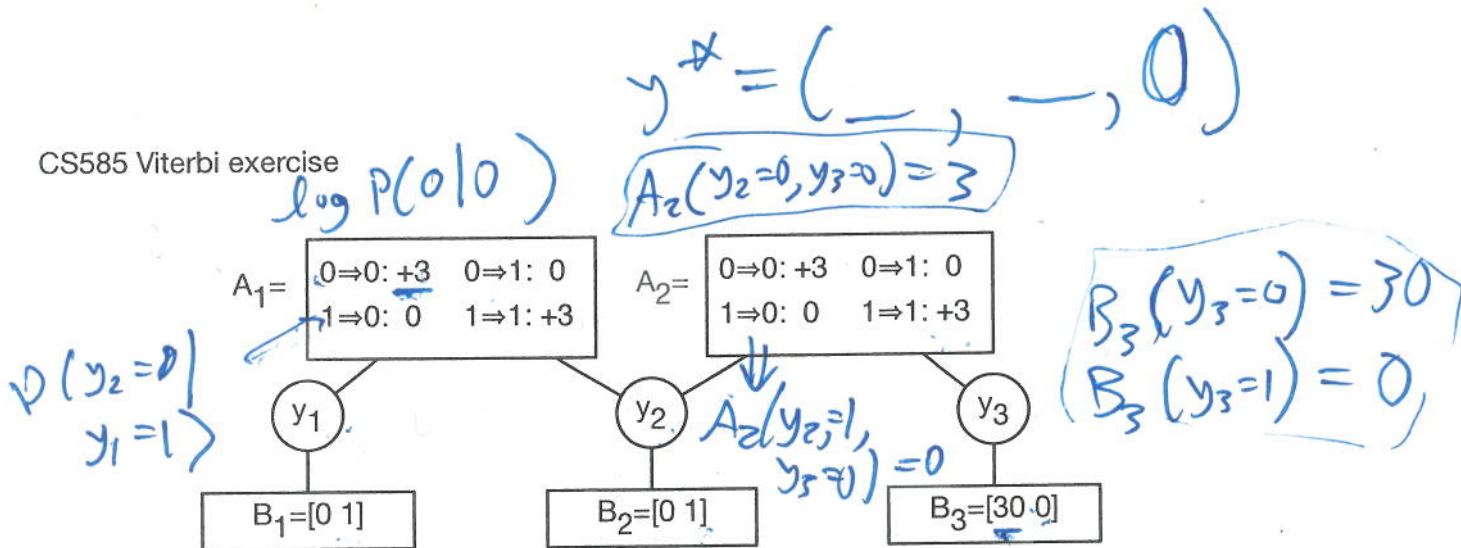
$$A(y_1, y_2) = \log P_{\text{trans}}(y_2 | y_1)$$

$$A(y_2, y_3) = \log P_{\text{trans}}(y_3 | y_2)$$

$$B_1(y_1) = \log P_{\text{emit}}(w_1 | y_1)$$

$$B_2(y_2) = \log P_{\text{emit}}(w_2 | y_2)$$

$$B_3(y_3) = \log P_{\text{emit}}(w_3 | y_3)$$



Sticky-favoring model over hidden state vocab {0,1}. Factor scores are "goodness points" are in log-scale additive form. (They're positive, though for an HMM they would all be negative.)

$$\log P(y | w) = (\text{constant}) + G(y_1, y_2, y_3)$$

$$G(y_1, y_2, y_3) = A(y_1, y_2) + A(y_2, y_3) + B_1(y_1) + B_2(y_2) + B_3(y_3)$$

Additive Viterbi

For  $t=1..T$ ,

For  $k$  in  $\{0,1\}$ ,

$$V_t[k] := \max_j (V_{t-1}[j] + A(j, k) + B_t(k))$$

$$B[k] := \arg \max_j (\dots)$$

For  $t=1$ , set  $A_0(\text{anything})=0$  and  $V_0(\text{anything})=0$

Final backtrace step: take best-scoring from last  $V_T$ , follow the backpointers all the way back

Run Viterbi and fill out the trellis with arcs like in the textbook's HMM example.

Most probable sequence  $y^* = ( 0 , 0 , 0 )$   $G(y^*) = 36$

START  
 $V_0(\text{START}) = 0$

