Homework 0: Probability Review and Survey

CS 585, UMass Amherst, Fall 2015

Last updated Sept 7; due Sept 10

Note

The reading for basic probability theory should help to figure out answers to these questions. Wikipedia is also pretty useful.

Turn in your answers to us at the start of lecture on Thursday. Either hand-written or typed up is fine (we prefer typed). Make sure to write your name. If your name is common, add your email address.

1 Domain of a joint distribution

1.1

A and B are discrete random variables. A could take on one of 4 possible values. B could take on one of 3 possible values. (In other words, the size of domain(A) is 4, and the size of domain(B) is 3.) How many possible outcomes does the joint distribution $P(A, B)$ define probabilities for?

2 Independence versus Basic Definitions

Say we have three random variables A and B and C. Note that we're using standard notation where $P(A, B) = P(B, A)$ (meaning, the joint probability of the joint event, the conjunction of A and B occurring). The MacKay reading is slightly different (he uses this notation to indicate ordering; we will come back to that later in the course).

2.1

Which of the following statements is true?

1. $P(A|B) = P(B|A)$
2. $P(A, B) = P(A|B)P(B)$
3. $P(A, B) = P(A)P(B)$
4. $P(A|B) = P(A)$
5. $P(A, B, C) = P(A)P(C)$
6. $P(A, B, C) = P(A)P(B)P(C)$

7. $P(A, B, C) = P(A)P(B|A)P(C|A, B)$
8. $P(A) = \sum_{b \in \text{domain}(B)} P(A, B = b)$
9. $P(A) = \sum_{b \in \text{domain}(B)} P(A|B = b)P(B = b)$

2.2

Now assume that A, B, and C are all independent of each other. Which of these statements is true?

3 Logarithms

3.1 Log-probs

Let p be a probability, so it is bounded to $[0, 1]$ (between 0 and 1, inclusive). What is the range of possible values for $\log(p)$?

3.2 Prob ratios

Let p and q both be probabilities. What is the range of possible values for p/q?

3.3 Log prob ratios

What is the range of possible values for $\log(p/q)$?

4 Deriving Bayes Rule

The definition of conditional probability can be written as $P(A, B) = P(A|B)P(B)$ or alternatively as $P(A|B) = P(A, B)/P(B)$. Starting from this, derive Bayes Rule, in this form:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

It should take only a few lines to prove/derive this. Note that you can apply the definition of conditional probability not just for $P(A, B)$ but also for $P(B, A)$. Here we are using standard notation where comma indicates conjunction/intersection so order of variables doesn't matter when defining a joint event.

5 Survey

5.1

What natural languages do you know? Which is your favorite?

5.2

What programming languages do you know? Which is your favorite?

5.3

What do you hope to get out of this course?