Lecture 15: Context-Free Grammars and the CKY algorithm

Intro to NLP, CS585, Fall 2014

http://people.cs.umass.edu/~brenocon/inlp2014/

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Includes material borrowed from Andrew McCallum, Noah Smith, Dan Klein, Chris Manning, Jurafsky&Martin

CMPSCI 585 In-class Exercise 10/28 ((actually 10/30	Name:	
	() () () () () () () () () ()		

Fill in the CYK dynamic programming table to parse the sentence below. In the bottom right corner, draw the two parse trees.

she	e eat	ts fish	n wit		op- cks
0	1	2	3	4	5
0	NP				
1					
2					
3					
4					

$S \rightarrow NP VP$	$NP \rightarrow she$
$NP \rightarrow NP PP$	$NP \rightarrow fish$
$VP \rightarrow V NP$	$NP \rightarrow fork$
$VP \rightarrow VP PP$	$NP \rightarrow chopsticks$
$PP \rightarrow P NP$	$V \rightarrow eats$
	$V \rightarrow fish$
	$P \rightarrow with$

Two views of syntax

• Constituents: phrase structure

Dependency structure

Parsing: applications

- Language modeling
 - John, who eats cookies, {love, loves} ...
- Machine translation
- Information extraction
- Grammar checking (MS Word!)
- Question answering
- NL interfaces to databases
- Sentiment analysis
- ...



Constituency (phrase structure)

- Phrase structure organizes words into nested constituents.
- How do we know what is a constituent? (Not that linguists don't argue about some cases.)
 - Distribution: a constituent behaves as a unit that can appear in different places:
 - John talked [to the children] [about drugs].
 - John talked [about drugs] [to the children].
 - *John talked drugs to the children about
 - Substitution/expansion/pro-forms:
 - I sat [on the box/right on top of the box/there].
 - Coordination, regular internal structure, no intrusion, fragments, semantics, ...

Ambiguity in parsing

- Syntactic ambiguity is widespread in language.
 - Attachment ambiguity
 - we ate sushi with chopsticks
 - I shot an elephant in my pajamas
 - Modifier scope
 - southern food store
 - etc.

Context-Free Grammars

- A generative formalism for constituency structures and text.
 - Generative view: produces a constit. tree and words in sentence.
 - Parsing view: given the words, what parse(s) could have generated it?
- Both boolean and probabilistic versions

Grammar (Production rules)

Lexicon

```
S \rightarrow NP VP
                                        |Det \rightarrow that | this | a
                                        Noun \rightarrow book \mid flight \mid meal \mid money
S \rightarrow Aux NP VP
S \rightarrow VP
                                        Verb \rightarrow book \mid include \mid prefer
NP \rightarrow Pronoun
                                        Pronoun \rightarrow I \mid she \mid me
NP \rightarrow Proper-Noun
                                        Proper-Noun \rightarrow Houston \mid TWA
NP \rightarrow Det Nominal
                                        Aux \rightarrow does
                                        Preposition \rightarrow from \mid to \mid on \mid near \mid through
Nominal \rightarrow Noun
Nominal \rightarrow Nominal Noun
Nominal \rightarrow Nominal PP
VP \rightarrow Verb
VP \rightarrow Verb NP
VP \rightarrow Verb NP PP
VP \rightarrow Verb PP
VP \rightarrow VP PP
PP \rightarrow Preposition NP
```

Figure 13.1 The \mathcal{L}_1 miniature English grammar and lexicon.

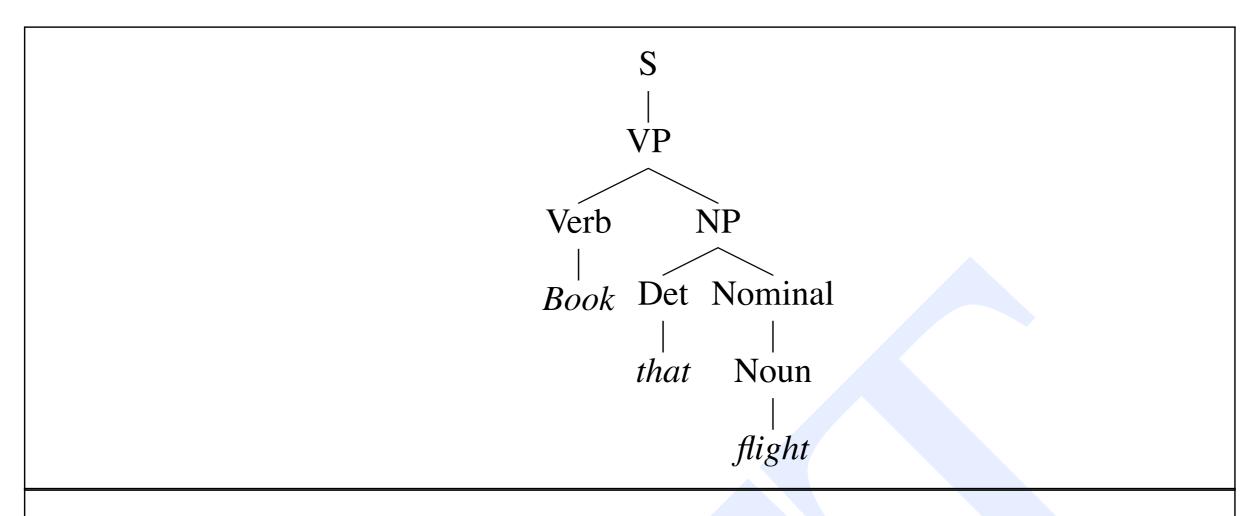


Figure 13.2 The parse tree for the sentence *Book that flight* according to grammar \mathcal{L}_1 .

Context-Free Grammars

- Unlike programming language grammars:
 Massive ambiguity!
- Unlike finite state grammars:
 Potentially infinite recursion

Computation/Statistics in NLP (in this course)

Mildly Context-Sensitive

Context Free Grammars

Finite State / Regular Languages

Independent Decisions

... CCG, TAG ... Recursive syntax (parsing) **CFG PCFG** [Today] [Thurs] Shallow syntax (Morph, POS, NER...) **HMM** MEMM, Regexes/ CRF/perc. **FSAs** Markov model: N-gram LM Logistic Reg. Naive Bayes Rule-based Generative Discrim. ~prob. model prob. model

Chomsky Hierarchy

Approaches to CFG parsing

- Top-down and Bottom-up search
- Shift-reduce: left-to-right
- [Today]
 Dynamic programming: CKY algorithm
 Exact search!
- Probabilistic/weighted variants of each of these:
 Find the best parse (e.g. most probable)

Binarized rules

Necessary for CKY algorithm

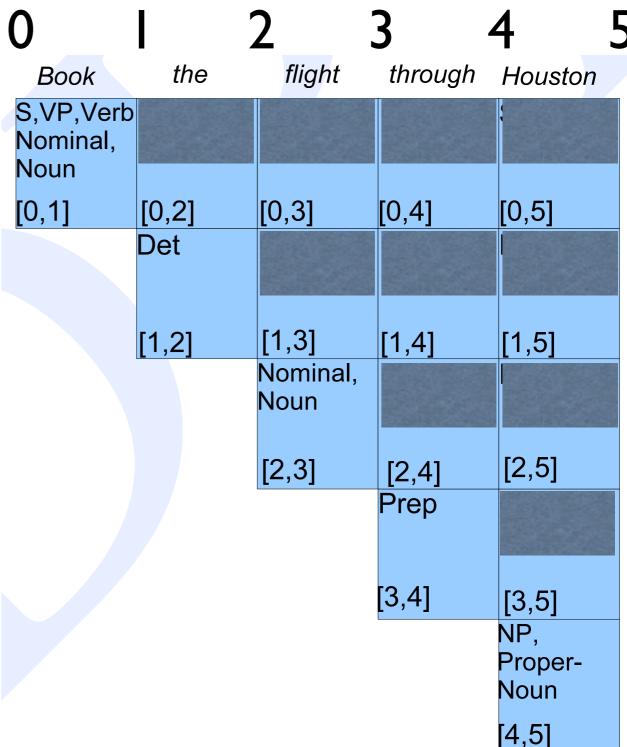
Can convert to equivalent binarized grammar

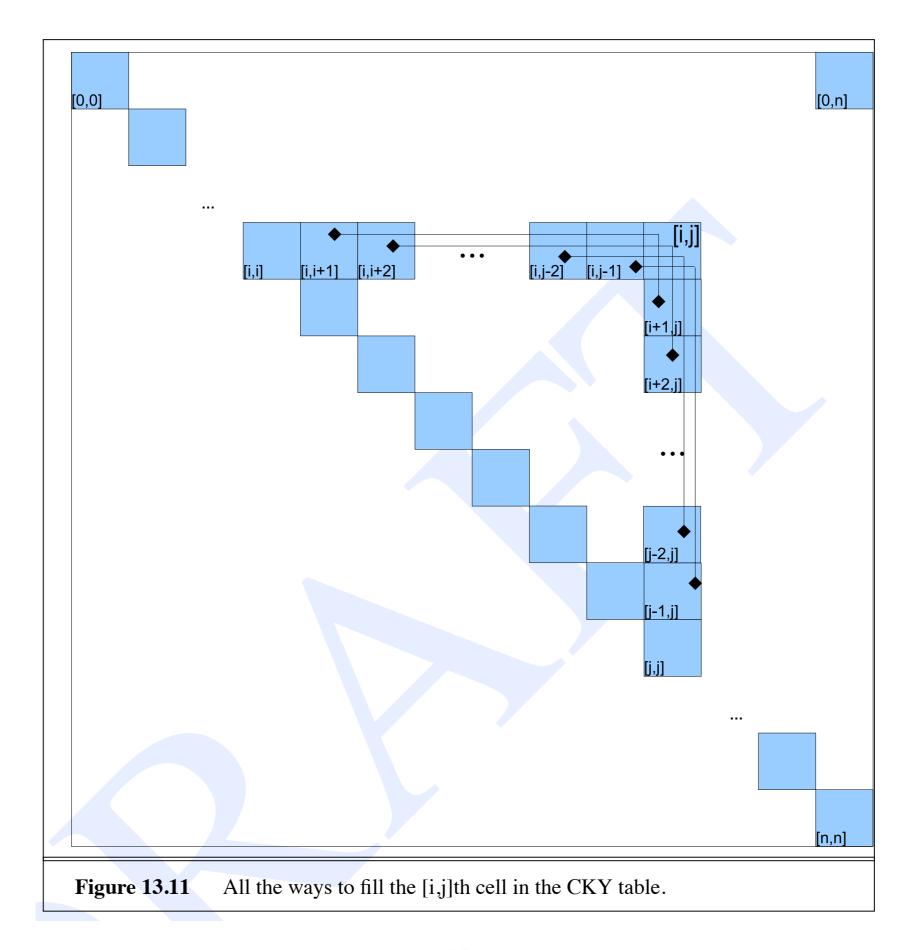
$$S \rightarrow NP VP$$

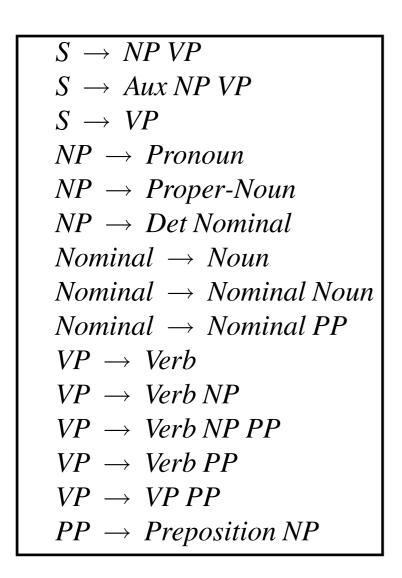
 $S \rightarrow Aux NP VP$

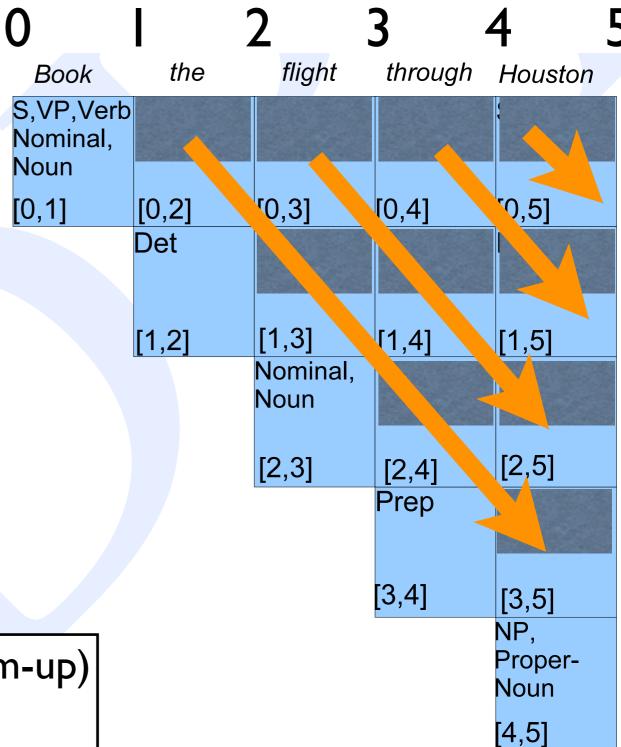
$$\begin{vmatrix}
S \rightarrow NP \ VP \\
S \rightarrow X1 \ VP \\
X1 \rightarrow Aux \ NP
\end{vmatrix}$$

- Fill in all length-1 spans with possible nonterminals.
- Go bottom-up: progressively fill each cell with possible states, based on possible combinations below.
- If the top cell [0,5] can expand from ROOT, then accept!
- To get one of possible parses: trace backpointers
- Dynamic programming: what's below the cell does not matter





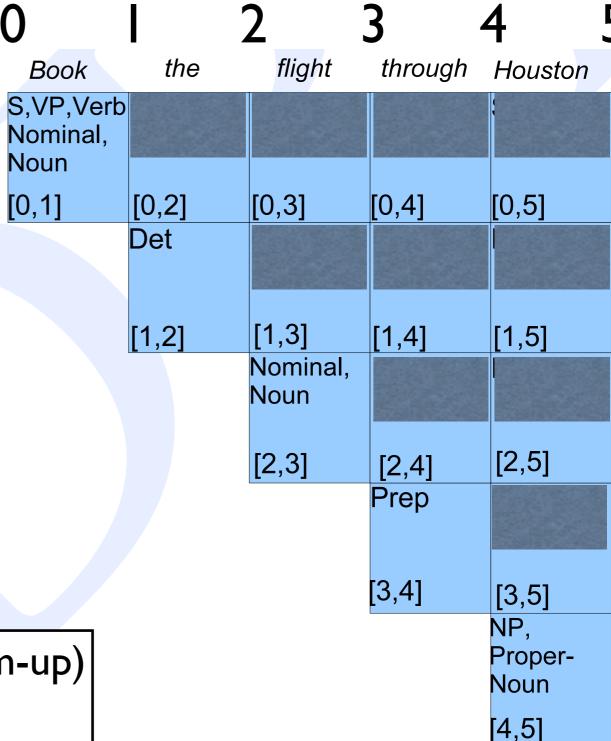




For cell [i,j] (loop through them bottom-up)
For possible splitpoint k=(i+1)..(j-1):
For every B in [i,k] and C in [k,j],
If exists rule A -> B C,

add A to cell [i,j]

S o NP VP S o Aux NP VP S o VP NP o Pronoun NP o Proper-Noun NP o Det Nominal Nominal o Noun Nominal o Nominal Noun Nominal o Nominal PP VP o Verb VP o Verb NP VP o Verb NP PP VP o Verb PP VP o VP o VP PPVP o VP PP



For cell [i,j] (loop through them bottom-up)
For possible splitpoint k=(i+1)..(j-1):
For every B in [i,k] and C in [k,j],
If exists rule A -> B C,

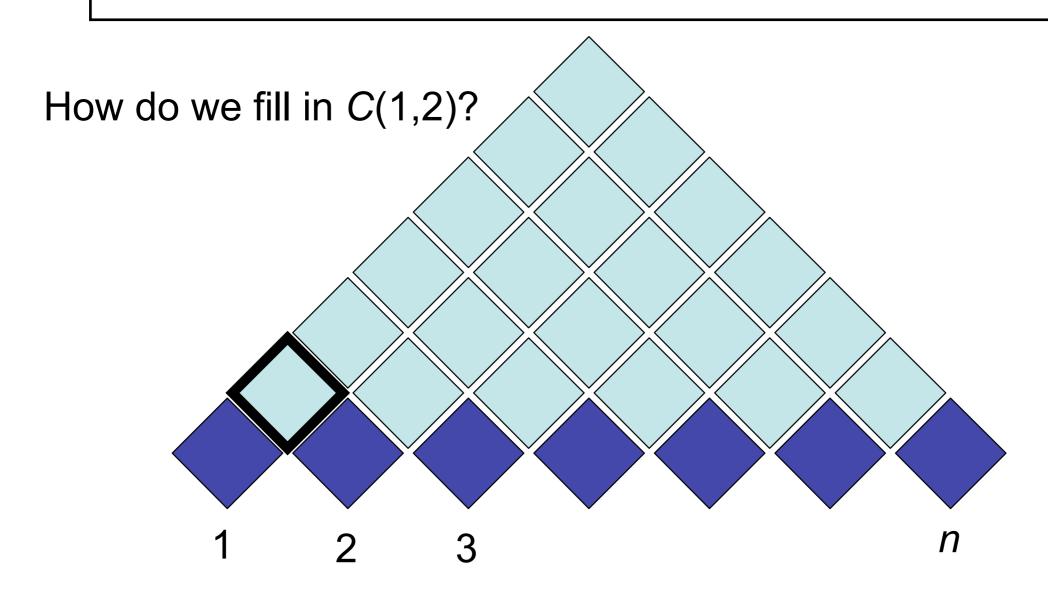
add A to cell [i,j]

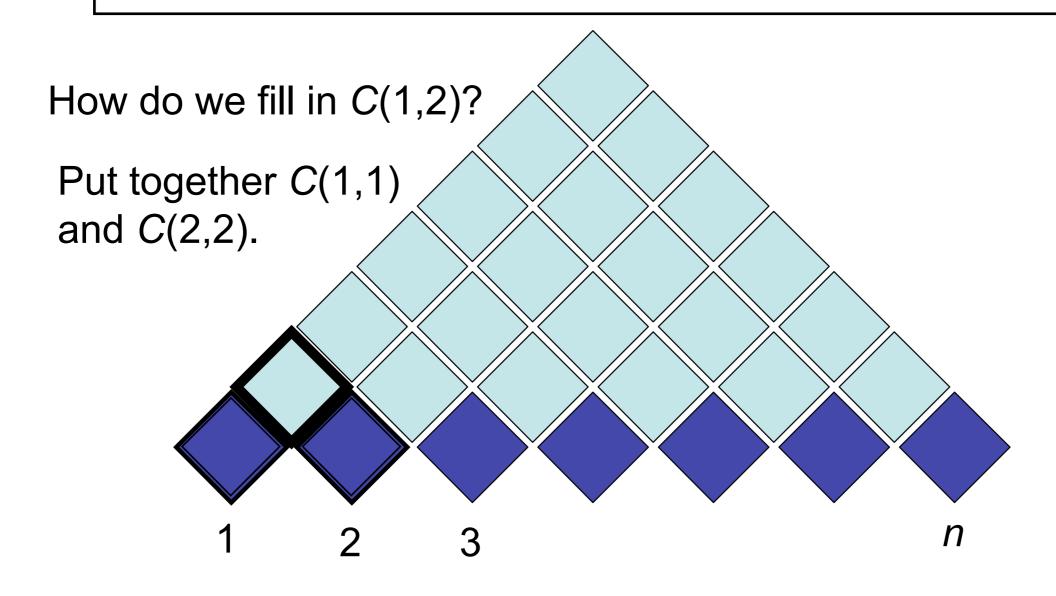
$S \rightarrow NP VP$
$S \rightarrow Aux NP VP$
$S \rightarrow VP$
$NP \rightarrow Pronoun$
NP ightarrow Proper-Noun
$NP \rightarrow Det\ Nominal$
Nominal ightarrow Noun
Nominal → Nominal Noun
$Nominal \rightarrow Nominal PP$
$VP \rightarrow Verb$
$VP \rightarrow Verb NP$
$VP \rightarrow Verb NP PP$
$VP \rightarrow Verb PP$
VP o VP PP
$PP \rightarrow Preposition NP$

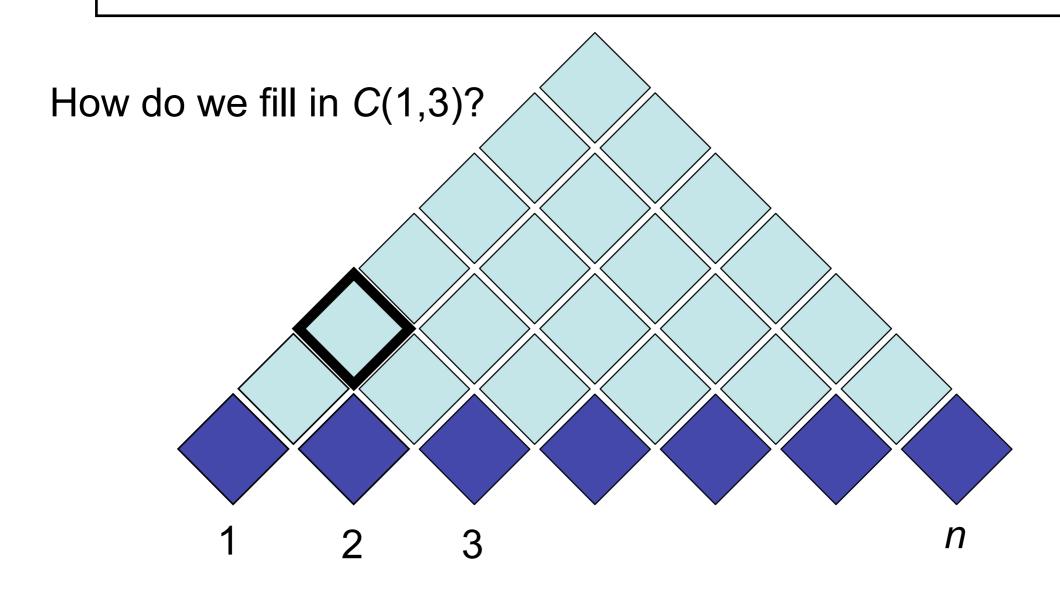
0		2 :	3	4 5
Book	the	flight	through	Houston
S,VP,Verb Nominal, Noun		S,VP,X2		S, VP
[0,1]	[0,2]	[0,3]	[0,4]	[0,5]
	Det	NP		NP
	[1,2]	[1,3]	[1,4]	[1,5]
		Nominal, Noun		Nominal
		[2,3]	[2,4]	[2,5]
			Prep	PP
			[3,4]	[3,5]
n-up)				NP, Proper- Noun
				[4,5]

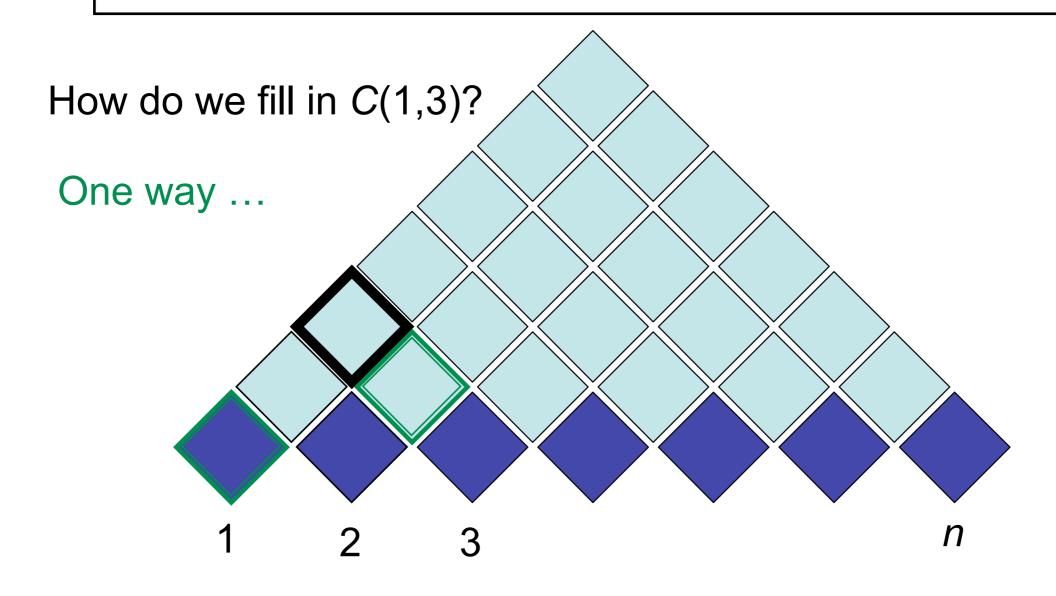
For cell [i,j] (loop through them bottom-up)
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If exists rule A -> B C,

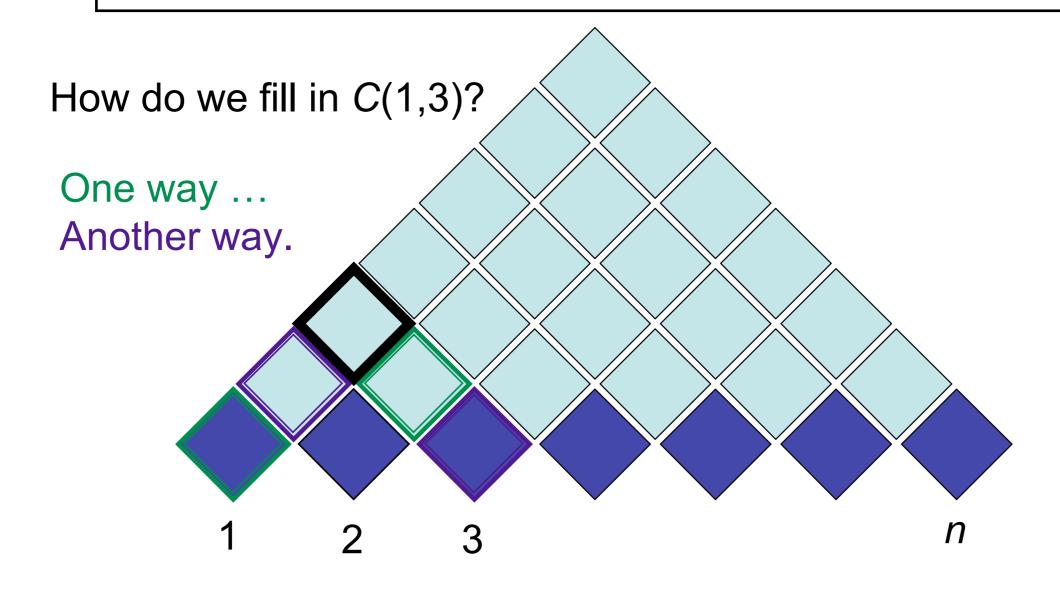
add A to cell [i,j]

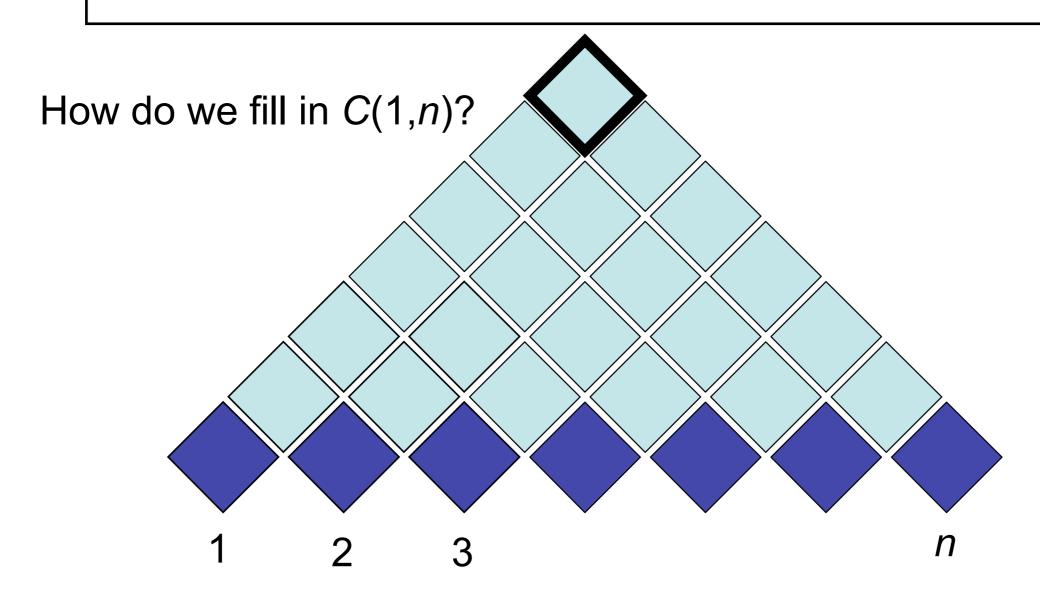


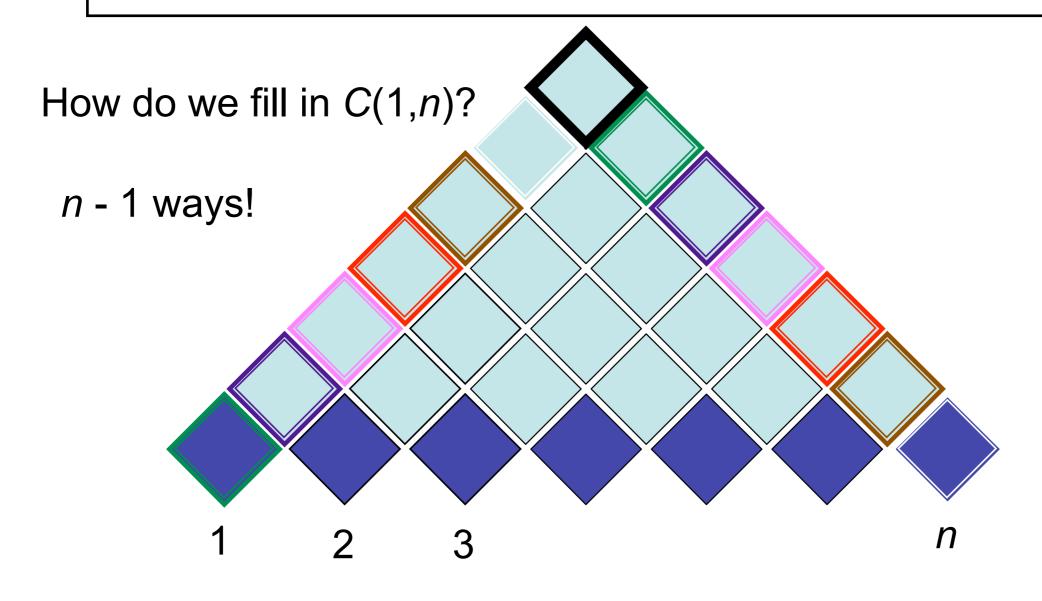




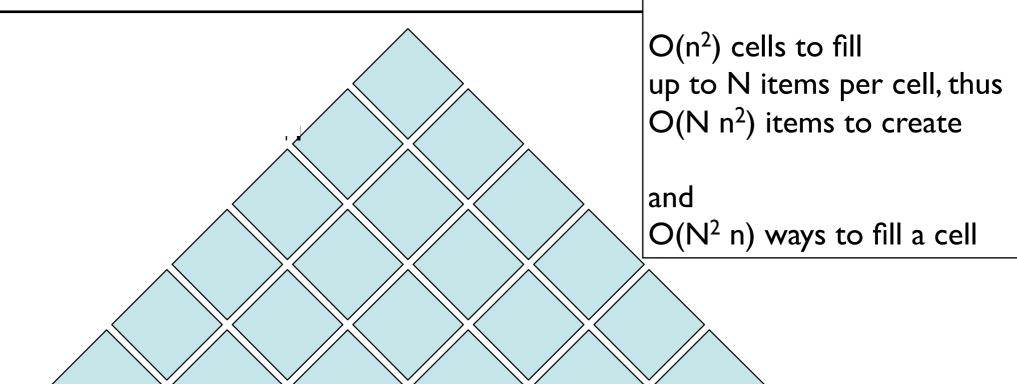








Where N is the number of nonterminals in the grammar, and n is the length of the sentence



Example with cost weights per expansion (Weighted CKY, a.k.a. Viterbi parsing)

time 1 flies 2 like 3 an 4 arrow 5

	NP	3								
	Vst	3								
0										
1			NP	4						
•			VP	4						
			\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	7						
2					P	2				
					\ V					
					V	J				
3							Det	1		
4									N	8

 $NP \rightarrow time$ $Vst \rightarrow time$ $NP \rightarrow flies$ $VP \rightarrow flies$ $P \rightarrow like$ $V \rightarrow like$ $V \rightarrow like$ $V \rightarrow an$ $V \rightarrow an$

1 S \rightarrow NP VP 6 S \rightarrow Vst NP 2 S \rightarrow S PP

1 $VP \rightarrow V NP$

 $2 \text{ VP} \rightarrow \text{VP PP}$

1 NP \rightarrow Det N

 $2 \text{ NP} \rightarrow \text{NP PP}$

3 NP \rightarrow NP NP

tim	ne 1	flies	2	like	3	an	4	arrow	5			
	NP	3										
	Vst	3										
0												
1			NP	4								
			VP	4								
2						P 2						
						V 5						
3								Det	1			
4										N	8	

 $6 S \rightarrow Vst NP$

 $2 S \rightarrow S PP$

1 $VP \rightarrow V NP$

 $2 \text{ VP} \rightarrow \text{VP PP}$

1 NP \rightarrow Det N

 $2 \text{ NP} \rightarrow \text{NP PP}$

3 NP \rightarrow NP NP

tim	ne 1	flies	2	like	3	a	n	4	arrow	5			
	NP	3	NP	10									
	Vst	3											
0													
0													
1			NP	4									
			VP	4									
2						P							
						V	5						
3									Det	1			
4											N	8	

 $6 S \rightarrow Vst NP$

 $2 S \rightarrow S PP$

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1 NP \rightarrow Det N

 $2 \text{ NP} \rightarrow \text{NP PP}$

 $3 \text{ NP} \rightarrow \text{NP NP}$

m	ie 1	l '	flies	2	like	3	i	an	4	a	rrow	5			
	NP		3	NP	10										
	Vst		3	S	8										
)															
				NP	4					+					
				VP	4										
<u> </u>							P	2							
								5							
3											Det	1			
<u> </u>													N	8	

 $6 S \rightarrow Vst NP$

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3 NP \rightarrow NP NP

tin	ne 1	flies	2	like	3	a	n	4	arrow	5			
	NP	3	NP	10									
	Vst	3	S	8									
			S	13									
0													
4			ND	4									
1			NP	4									
			VP	4									
2						P	2						
						V	5						
3									Det	1			
4											N	8	

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3 NP \rightarrow NP NP

ne 1	Tiles	2	like	3		an	4	arrow	5			
NP	3	NP	10									
Vst	3	S	8									
		S	13									
					_							
		VP	4									
					P	2		_				
					V	5						
								Det	1			
										N	8	
	NP	NP 3	NP 3 NP Vst 3 S	NP 3 NP 10 S 8 S 13 NP 4	NP 3 NP 10 S 8 S 13	NP 3 NP 10 S 8 S 13 NP 4 P	NP 3 NP 10 S 8 S 13 NP 4 P 2	NP 3	NP 3	NP 3	NP 3	NP 3 NP 10 S 8 S 13 NP 4 P 2 P 2 V 5 Det 1

 $6 S \rightarrow Vst NP$

 $2 S \rightarrow S PP$

1 $VP \rightarrow V NP$

 $2 \text{ VP} \rightarrow \text{VP PP}$

1 NP \rightarrow Det N

 $2 \text{ NP} \rightarrow \text{NP PP}$

3 NP \rightarrow NP NP

flies 2 like time 1 3 an 4 arrow 5 NP 3 NP 10 Vst 3 S 8 S 13 0 NP 4 **VP** 4 P 2 5 V 3 NP 10 Det 4 N 8

1 S \rightarrow NP VP 6 S \rightarrow Vst NP

 $2 S \rightarrow S PP$

1 $VP \rightarrow V NP$

 $2 \text{ VP} \rightarrow \text{VP PP}$

1 NP \rightarrow Det N

 $2 \text{ NP} \rightarrow \text{NP PP}$

3 NP \rightarrow NP NP

time 1 flies 2 like 3 an 4 arrow 5

	NP	3	NP	10						
	Vst	3	S	8						
			S	13						
0										
1			NP	4	_					
			VP	4						
2					Р	2	_			
					V	5				
3							Det	1	NP	10
4									N	8

 $1 S \rightarrow NP VP$

 $6 S \rightarrow Vst NP$

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3 $NP \rightarrow NP NP$

time 1 flies 2 like 3 an 4 arrow 5

	NP	3	NP	10	_			
	Vst	3	S	8				
			S	13				
0								
1			NP	4	_	_		
			VP	4				
2					P 2	_	PP	12
					V 5			
3						Det 1	NP	10
4							N	8

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 $2 \text{ NP} \rightarrow \text{NP PP}$

3 NP \rightarrow NP NP

	NP	3	NP	10	_					
	Vst	3	S	8						
			S	13						
0										
1			NP	4	_		_			
			VP	4						
2					Р	2	_		PP	12
					V	5	-		VP	16
3							Det	1	NP	10
							Det	<u> </u>		
4									N	8

 $1 S \rightarrow NP VP$

 $6 S \rightarrow Vst NP$

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 $2 \text{ NP} \rightarrow \text{NP PP}$

3 NP \rightarrow NP NP

	NP	3	NP	10	_					
	Vst	3	S	8						
			S	13						
0										
1			NP	4	_		_			
			VP	4						
2					Р	2	_		PP	12
					V	5			VP	16
3							Det	1	NP	10
4									N	8

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1 NP \rightarrow Det N

 $2 \text{ NP} \rightarrow \text{NP PP}$

3 $NP \rightarrow NP NP$

0	NP Vst	3 3	NP S S	10 8 13	_					
1			NP VP	4 4	_		_		NP	18
2					P V	2 5	_		PP VP	12 16
3							Det	1	NP	10
4									N	8

 $1 S \rightarrow NP VP$

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3 NP \rightarrow NP NP

0	NP Vst	3 3	NP S S	10 8 13	_					
1			NP VP	4 4	_		_		NP S	18 21
2					P V	2 5	_		PP VP	12 16
3							Det	1	NP	10
4									N	8

 $1 S \rightarrow NP VP$

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 $2 \text{ NP} \rightarrow \text{NP PP}$

3 NP \rightarrow NP NP

0	NP Vst	3 3	NP S S	10 8 13	_					
1			NP VP	4 4	_		_		NP S VP	18 21 18
2					P V	2 5	_		PP VP	12 16
3							Det	1	NP	10
4									N	8

 $1 S \rightarrow NP VP$

 $6 S \rightarrow Vst NP$

 $2 S \rightarrow S PP$

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 $2 \text{ VP} \rightarrow \text{VP PP}$

1 NP \rightarrow Det N

 $2 \text{ NP} \rightarrow \text{NP PP}$

3 NP \rightarrow NP NP

time	1	flies	2	like	3	an	4	arrow	5
	•					an		allow	U

	NP	3	NP	10	_		_			
	Vst	3	S	8						
			S	13						
0										
1			NP	4	_		_		NP	18
			VP	4					S	21
									VP	18
2					Р	2	_		PP	12
					V	5			VP	16
3							Det	1	NP	10
4									N	8

 $6 S \rightarrow Vst NP$

 $2 S \rightarrow S PP$

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 $2 \text{ VP} \rightarrow \text{VP PP}$

1 NP \rightarrow Det N

 $2 \text{ NP} \rightarrow \text{NP PP}$

3 NP \rightarrow NP NP

time	1	flies	2	like	3	an	4	arrow	5
	-	11100							

NP	3	NP	10	_		_		NP	24
Vst	3	S	8						
		S	13						
		NP	4	_		_		NP	18
		VP	4					S	21
								VP	18
				Р	2	_		PP	12
				V	5			VP	16
						Det	1	NP	10
								N	8
			Vst 3 SSS	Vst 3 S 8 S 13	Vst 3 S 8 S 13 NP 4 P	Vst 3	Vst 3	Vst 3	Vst 3

$$1 S \rightarrow NP VP$$

$$6 S \rightarrow Vst NP$$

$$2 S \rightarrow S PP$$

1
$$VP \rightarrow V NP$$

$$2 \text{ VP} \rightarrow \text{VP PP}$$

1 NP
$$\rightarrow$$
 Det N

$$2 \text{ NP} \rightarrow \text{NP PP}$$

$$3 \text{ NP} \rightarrow \text{NP NP}$$

$$0 PP \rightarrow PNP$$

time	1	flies	2	like	3	an	4	arrow	5
				IIIC		an		arrow	J

	NP	3	NP	10	_		_		NP	24
	Vst	3	S	8					S	22
			S	13						
0										
1			NP	4			_		NP	18
			VP	4					S	21
									VP	18
2					Р	2	_		PP	12
					V	5			VP	16
3							Det	1	NP	10
4									N	8

 $6 S \rightarrow Vst NP$

 $2 S \rightarrow S PP$

1 $VP \rightarrow V NP$

 $2 \text{ VP} \rightarrow \text{VP PP}$

1 NP \rightarrow Det N

 $2 \text{ NP} \rightarrow \text{NP PP}$

3 NP \rightarrow NP NP

4.1	_	CLI		444			_		_
time	1	tlies	7	like	3	an	4	arrow	5
		11163				an		arrow	J

	NP	3	NP	10	_		_		NP	24
	Vst	3	S	8					S	22
			S	13					S	27
0										
1			NP	4	_		_		NP	18
			VP	4					S	21
									VP	18
2					Р	2	_		PP	12
					V	5			VP	16
3							Det	1	NP	10
4									N	8
4									N	8

$$1 S \rightarrow NP VP$$

$$6 S \rightarrow Vst NP$$

$$2 S \rightarrow S PP$$

1
$$VP \rightarrow V NP$$

$$2 \text{ VP} \rightarrow \text{VP PP}$$

1 NP
$$\rightarrow$$
 Det N

$$2 \text{ NP} \rightarrow \text{NP PP}$$

3 NP
$$\rightarrow$$
 NP NP

$$0 PP \rightarrow PNP$$

time	1	flies	2	like	3	an	4	arrow	5
uiiie		11162				all	4	alluw	J

	NP	3	NP	10	_		_		NP	24
	Vst	3	S	8					S	22
			S	13					S	27
0										
					1					
1			NP	4	_		_		NP	18
			VP	4					S	21
									VP	18
2					Р	2	_		PP	12
					V	5			VP	16
3							Det	1	NP	10
4									N	8

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$$6 S \rightarrow Vst NP$$

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$$VP \rightarrow V NP$$

$$2 \text{ VP} \rightarrow \text{VP PP}$$

1 NP
$$\rightarrow$$
 Det N

$$2 \text{ NP} \rightarrow \text{NP PP}$$

3 NP
$$\rightarrow$$
 NP NP

$$0 PP \rightarrow PNP$$

time 1 flies 2 like 3 an 4 arrow	5
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	NP	3	NP	10	_		_		NP	24
	Vst	3	S	8					S	22
			S	13					S	27
0									NP	24
1			NP	4			_		NP	18
			VP	4					S	21
									VP	18
2					P 2	2	_		PP	12
					V 5	5			VP	16
3							Det	1	NP	10
4									N	8

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time	1	flies	2	like	3	an	4	arrow	5
				IIIC		an		anov	J

	NP	3	NP	10	_		_		NP	24
	Vst	3	S	8					S	22
			S	13					S	27
0									NP	24
									S	27
1			NP	4	_		_		NP	18
			VP	4					S	21
									VP	18
2					Р	2	_		PP	12
					V	5			VP	16
3							Det	1	NP	10
4									N	8

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 $2 S \rightarrow S PP$

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1 NP \rightarrow Det N

 $2 \text{ NP} \rightarrow \text{NP PP}$

3 NP \rightarrow NP NP

4.1	4	CLI		100			_		_
time	1	tlies	2	like	3	an	4	arrow	5
						a i i		CIT O VV	

	NP	3	NP	10	_		_		NP	24
	Vst	3	S	8					S	22
			S	13					S	27
0									NP	24
									S	27
									S	22
1			NP	4			_		NP	18
			VP	4					S	21
									VP	18
2					Р	2	_		PP	12
					V	5			VP	16
3							Det	1	NP	10
4									N	8

$$1 S \rightarrow NP VP$$

$$6 S \rightarrow Vst NP$$

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$$VP \rightarrow V NP$$

$$2 \text{ VP} \rightarrow \text{VP PP}$$

1 NP
$$\rightarrow$$
 Det N

$$2 \text{ NP} \rightarrow \text{NP PP}$$

3 NP
$$\rightarrow$$
 NP NP

$$0 PP \rightarrow PNP$$

time 1 flies 2 like 3 an 4 arrow 5	time	1	flies	2	like	3	an	4	arrow	5
------------------------------------	------	---	-------	---	------	---	----	---	-------	---

	NP	3	NP	10	_		_		NP	24
	Vst	3	S	8					S	22
			S	13					S	27
0									NP	24
									S	27
									S	22
									S	27
1			NP	4	_		_		NP	18
			VP	4					S	21
									VP	18
2					Р	2	_		PP	12
					V	5			VP	16
3							Det	1	NP	10
4									N	8

$$1 S \rightarrow NP VP$$

$$6 S \rightarrow Vst NP$$

$$2 S \rightarrow S PP$$

1
$$VP \rightarrow V NP$$

$$2 \text{ VP} \rightarrow \text{VP PP}$$

1 NP
$$\rightarrow$$
 Det N

$$2 \text{ NP} \rightarrow \text{NP PP}$$

3 NP
$$\rightarrow$$
 NP NP

$$0 PP \rightarrow PNP$$

S

Follow backpointers

4 arrow

Det 1

NP

N

10

8

an

like 3

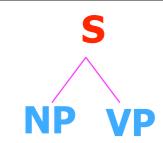
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	NP	3	NP	10	_		_	NP	24
	Vst	3	S	8				S	22
			S	13				S	27
0								NP	24
								S	27
								S	22
								S	27
1			NP	4	_		_	NP	18
			VP	4				S	21
								VP	18
2					Р	2	_	PP	12
					V	5		VP	16

- $1 S \rightarrow NP VP$
- 6 S \rightarrow Vst NP
- $2 S \rightarrow S PP$
- 1 $VP \rightarrow V NP$
- $2 \text{ VP} \rightarrow \text{VP PP}$
- 1 NP \rightarrow Det N
- $2 \text{ NP} \rightarrow \text{NP PP}$
- 3 NP \rightarrow NP NP
- $0 \text{ PP} \rightarrow \text{P NP}$

3

4

time 1 flies 2



	_		_		_		_		
time	1	flies	2	like	3	an	4	arrow	5
						an		allow	

	NP	3	NP	10	_		_		NP	24
	Vst	3	S	8					S	22
			S	13					S	27
0									NP	24
									S	27
									S	22
									S	27
1			NP	4	_		_		NP	18
			VP	4					S	21
									VP	18
2					Р	2	_		PP	12
					V	5			VP	16
3							Det	1	NP	10
4									N	8

6 S \rightarrow Vst NP

 $2 S \rightarrow S PP$

1 $VP \rightarrow V NP$

2 $VP \rightarrow VP PP$

1 NP \rightarrow Det N

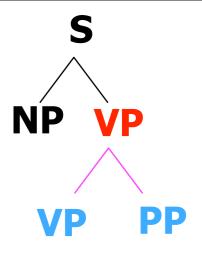
2 NP \rightarrow NP PP

3 NP \rightarrow NP NP

 $0 \text{ PP} \rightarrow \text{P NP}$



	NP	3	NP	10		_		NP	24
	Vst	3	S	8				S	22
			S	13				S	27
0								NP	24
								S	27
								S	22
								S	27
1			NP	4	_	_		NP	18
			VP	4				S	21
								VP	18
2					P 2	_		PP	12
					V 5			VP	16
3						Det	1	NP	10
4								N	8



6 S \rightarrow Vst NP

 $2 S \rightarrow S PP$

1 $VP \rightarrow V NP$

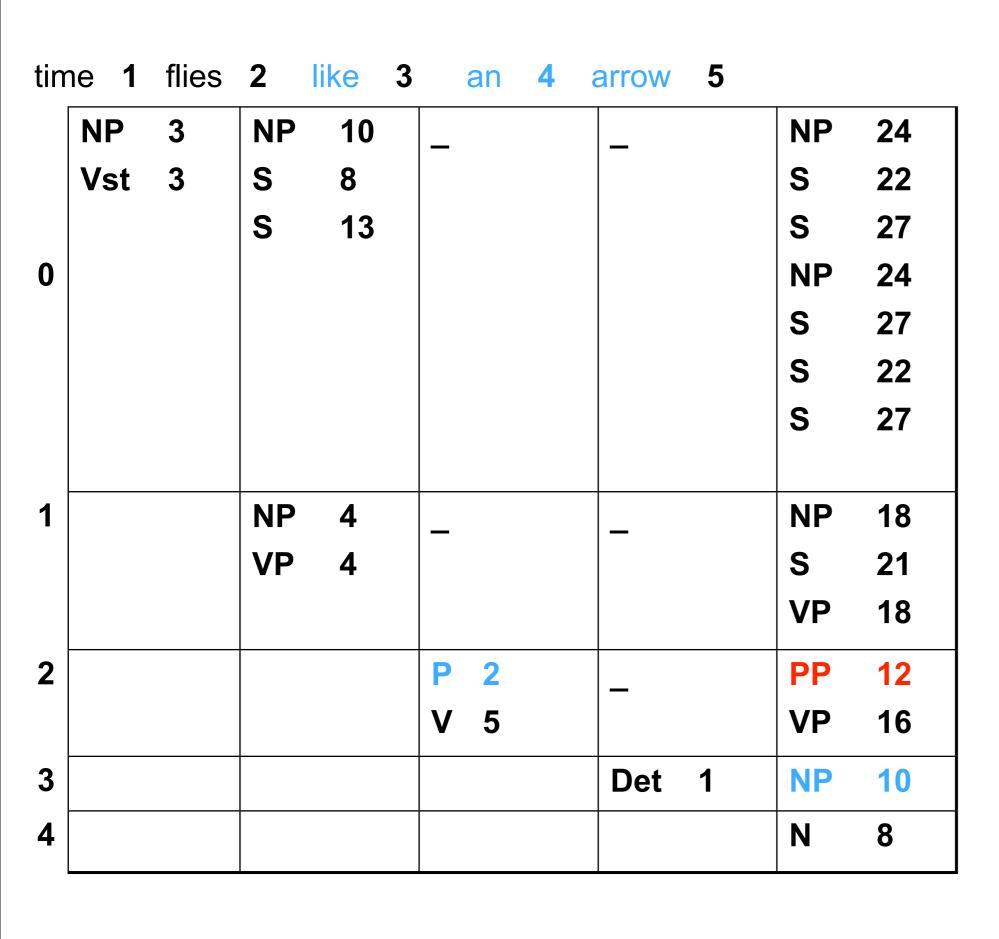
 $2 \text{ VP} \rightarrow \text{VP PP}$

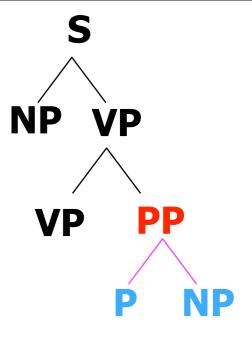
1 NP \rightarrow Det N

2 NP \rightarrow NP PP

3 NP \rightarrow NP NP

 $0 \text{ PP} \rightarrow \text{P NP}$





 $6 S \rightarrow Vst NP$

 $2 S \rightarrow S PP$

1 $VP \rightarrow V NP$

2 $VP \rightarrow VP PP$

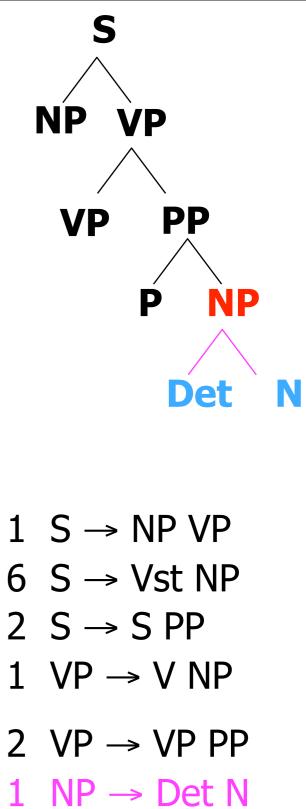
1 NP \rightarrow Det N

2 NP \rightarrow NP PP

3 NP \rightarrow NP NP

 $0 \text{ PP} \rightarrow \text{P NP}$

tim	ne	1	flies	2	like	3		an	4	arrow	5		
	NF)	3	NP	10		_			_		NP	24
	Vs	st	3	S	8							S	22
				S	13							S	27
0												NP	24
												S	27
												S	22
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1				NP	4		_			_		NP	18
				VP	4							S	21
												VP	18
2							P	2		_		PP	12
							V	5				VP	16
3										Det	1	NP	10
4												N	8
				<u>I</u>						I		ı	



$$2 \text{ VP} \rightarrow \text{VP PP}$$

1 NP
$$\rightarrow$$
 Det N

2 NP
$$\rightarrow$$
 NP PP

3 NP
$$\rightarrow$$
 NP NP

$$0 \text{ PP} \rightarrow \text{P NP}$$