From J&M chapter 7 -- Jason Eisner's
ice cream / weather HMM example.

This is Eisner ----> (Associate Professor, John Hopkins University)

Model

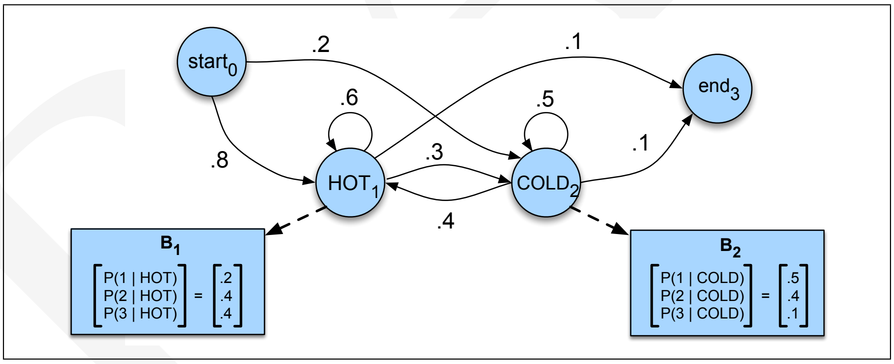


Figure 7.3 A hidden Markov model for relating numbers of ice creams eaten by Jason (the observations) to the weather (H or C, the hidden variables).

Viterbi algorithm

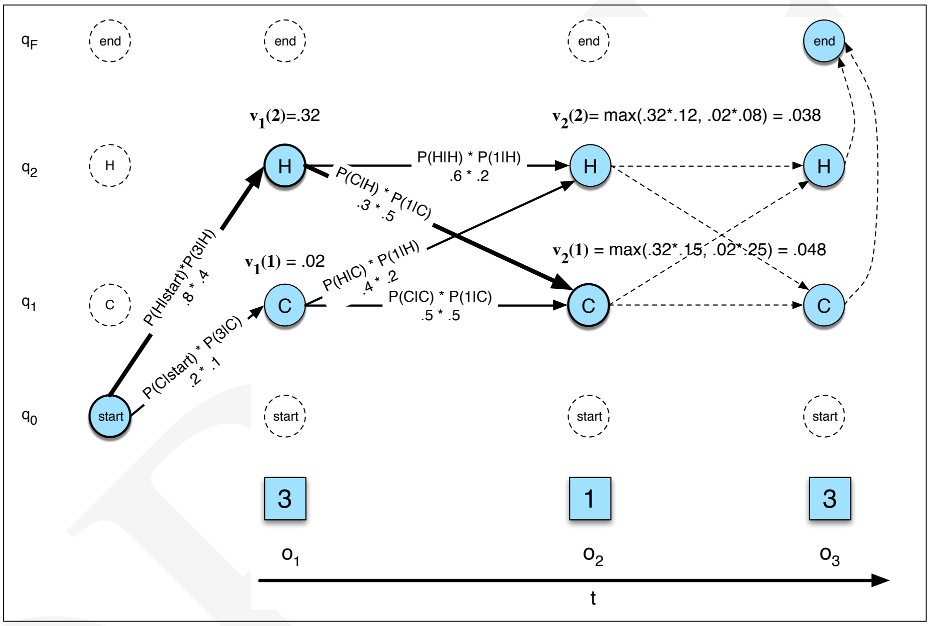
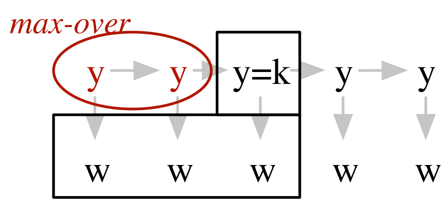


Figure 7.10 The Viterbi trellis for computing the best path through the hidden state space for the ice-cream eating events *3 1 3*. Hidden states are in circles, observations in squares. White (unfilled) circles indicate illegal transitions. The figure shows the computation of $v_t(j)$ for two states at two time steps. The computation in each cell follows Eq. 7.19: $v_t(j) = \max_{1 \leq i \leq N-1} v_{t-1}(i)\ a_{ij}\ b_j(o_t)$. The resulting probability expressed in each cell is Eq. 7.18: $v_t(j) = P(q_0, q_1, \ldots, q_{t-1}, o_1, o_2, \ldots, o_t, q_t = j|\lambda)$.



Declaratively:

$$V_t[k] = \max_{y_1 \cdots y_{t-1}} P(y_t = k, \quad y_1..y_{t-1}, w_1..w_t)$$

Algorithm, for each *t=1..N,*

$$V_t[k] := \max_{j=1..K} \Big(V_{t-1}[j]\ P_{trans}(j \to k)\ P_{emit}(w_t|k) \Big)$$

Forward algorithm

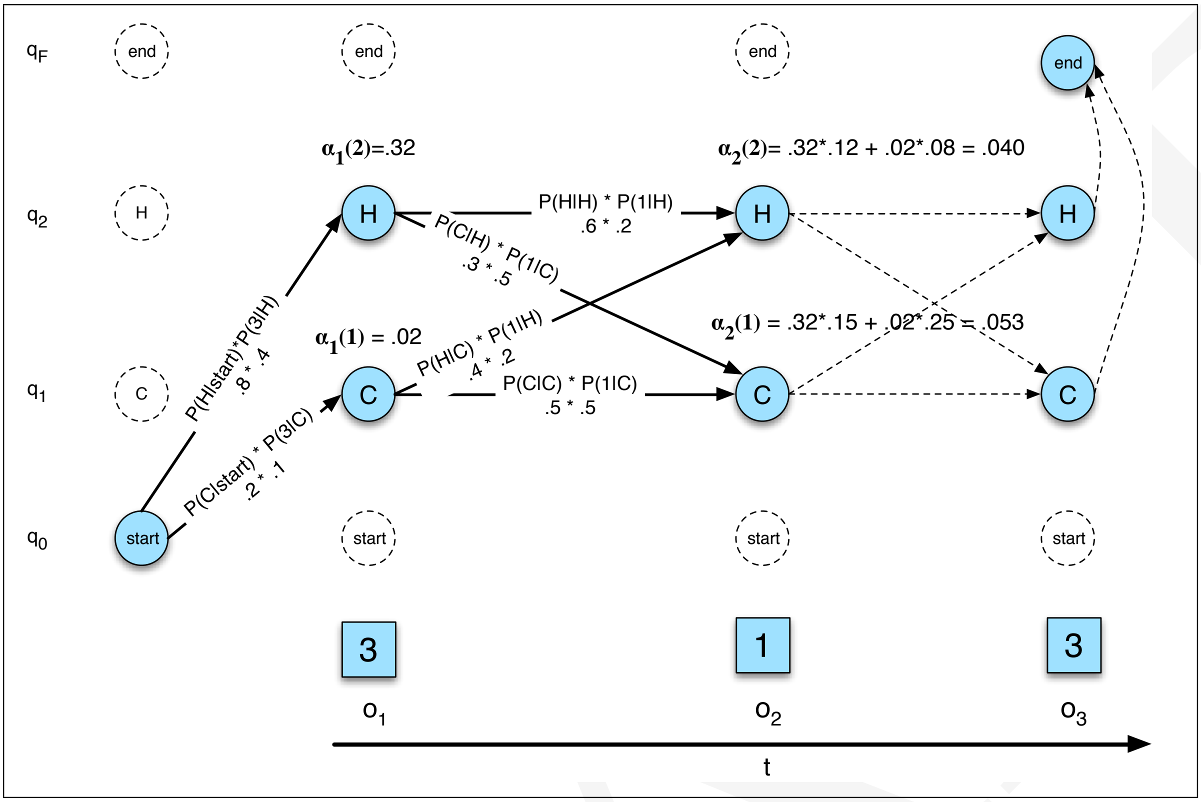
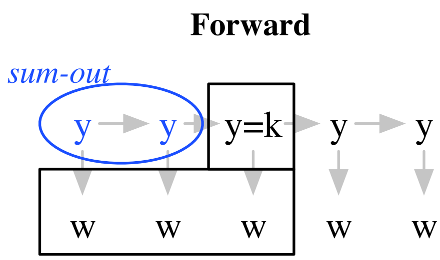


Figure 7.7 The forward trellis for computing the total observation likelihood for the ice-cream events *3 1 3*. Hidden states are in circles, observations in squares. White (unfilled) circles indicate illegal transitions. The figure shows the computation of $\alpha_t(j)$ for two states at two time steps. The computation in each cell follows Eq. 7.14: $\alpha_t(j) = \sum_{i=1}^{N} \alpha_{t-1}(i) a_{ij} b_j(o_t)$. The resulting probability expressed in each cell is Eq. 7.13: $\alpha_t(j) = P(o_1, o_2 \ldots o_t, q_t = j|\lambda)$.



Declaratively:

$$\alpha_t[k] = \sum_{y_1 \ldots y_t} P(y_t = k,\ w_1..w_t,\ y_1..y_{t-1})$$

Algorithm, for each *t=1..N,*

$$\alpha_t[k] := \sum_{j=1..K} \left(\alpha_{t-1}[j]\ P_{trans}(j \to k)\ P_{emit}(w_t|k) \right)$$