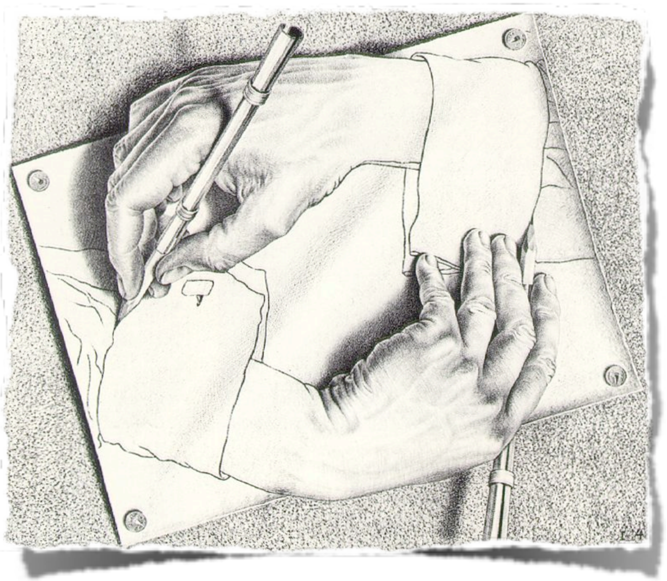
Declaring Independence via the Sketching of Sketches



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Until August '08 -- Hire Me!

The Problem

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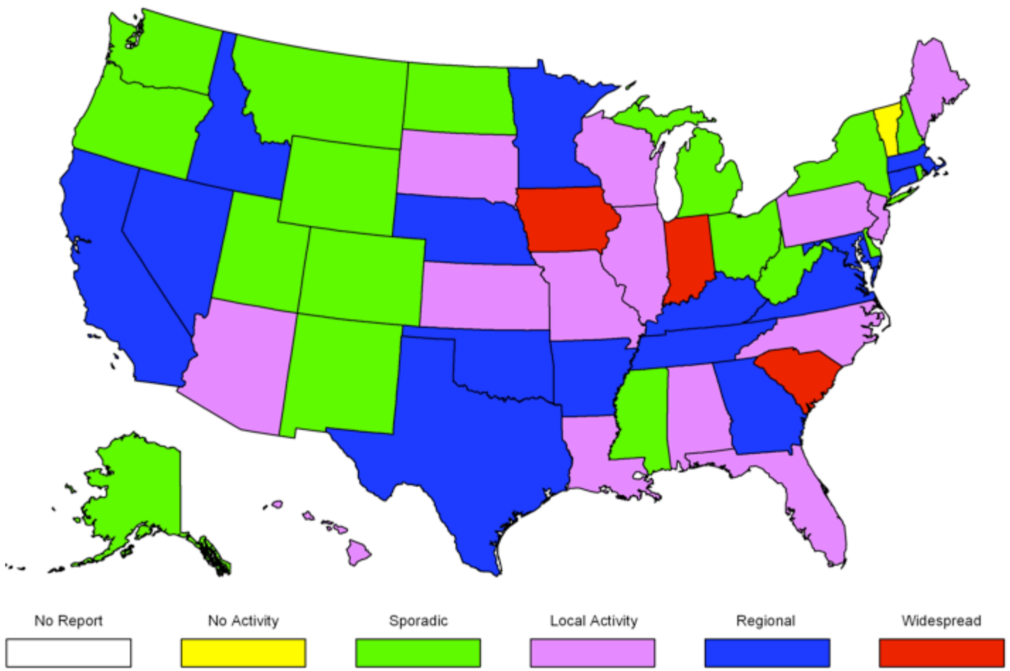

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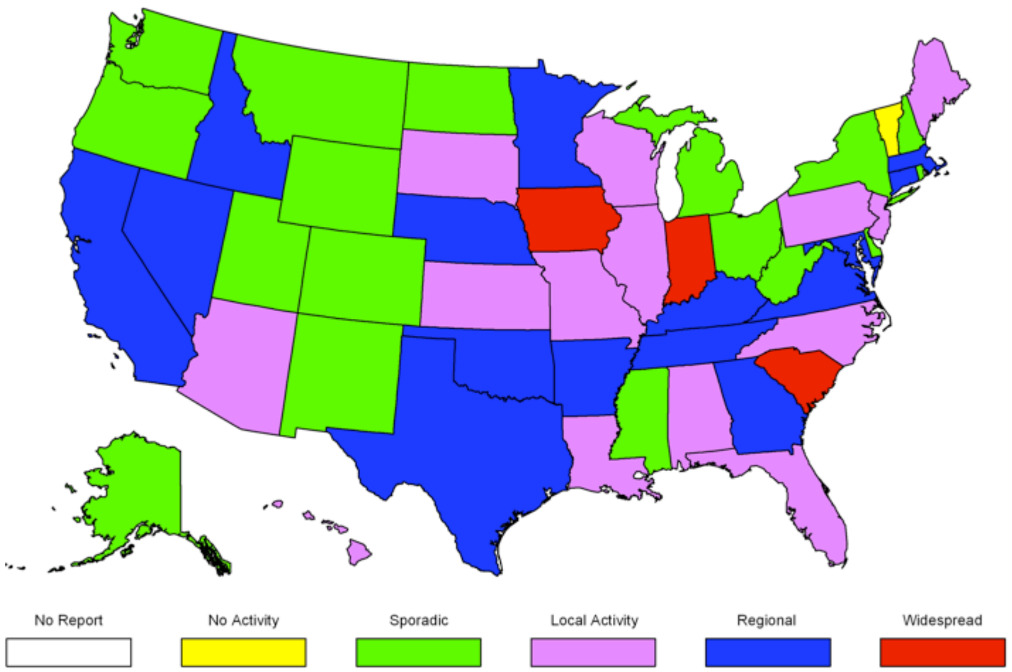

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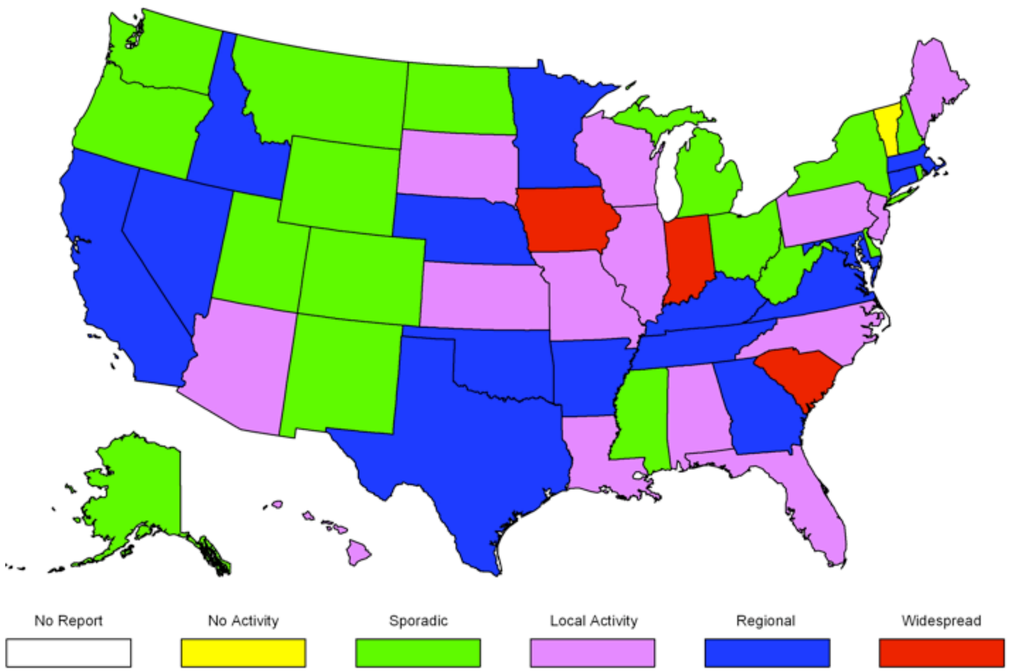

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- *Stream (sub-linear space):*
  Access pairs sequentially or "online" and limited memory.

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  Marginals: $(p_1, \ldots, p_n)$, $(q_1, \ldots, q_n)$

  Joint: $(r_{11}, r_{12}, \ldots, r_{nn})$

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$$L_1(s-r) = \sum_{i,j} |s_{ij} - r_{ij}|$$

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- *Previous work:* Can estimate L_1 *and* L_2 between marginals.

  [Alon, Matias, Szegedy '96], [Feigenbaum et al. '99], [Indyk '00], [Guha, Indyk, McGregor '07], [Ganguly, Cormode '07]

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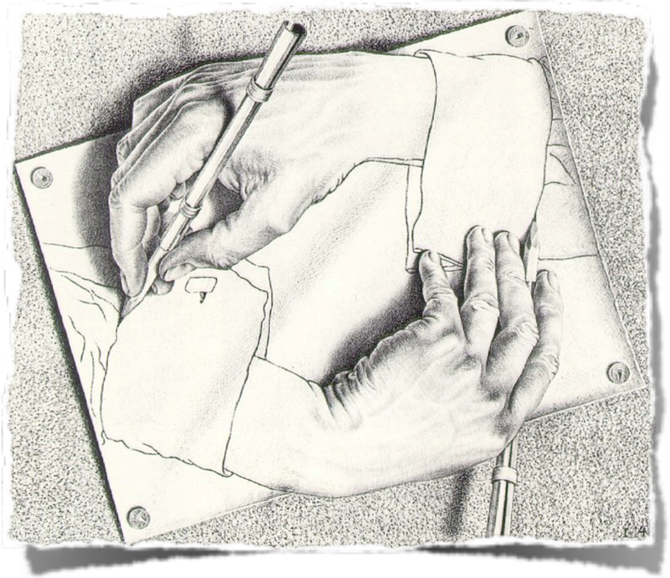
- ***Other Results:***

  $L_1(s-r)$: Additive approximations

  Mutual Information: Additive but not $(1+\epsilon)$-factor approx.

  Distributed Model: Pairs are observed by different parties.

a) Neat Result for L_2
b) Sketching Sketches
c) Other Results



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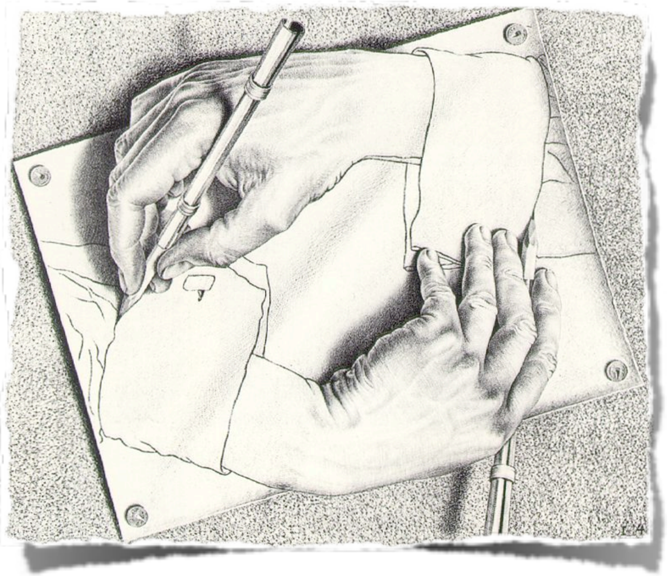

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- *Bad News:* z is no longer 4-wise independent even if *x* and *y* are fully random, e.g.,

$$z_{11} z_{12} z_{21} z_{22} = (x_1)^2 (x_2)^2 (y_1)^2 (y_2)^2 = 1$$

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- Less independence useful for range-sums. [Rusu, Dobra '06]

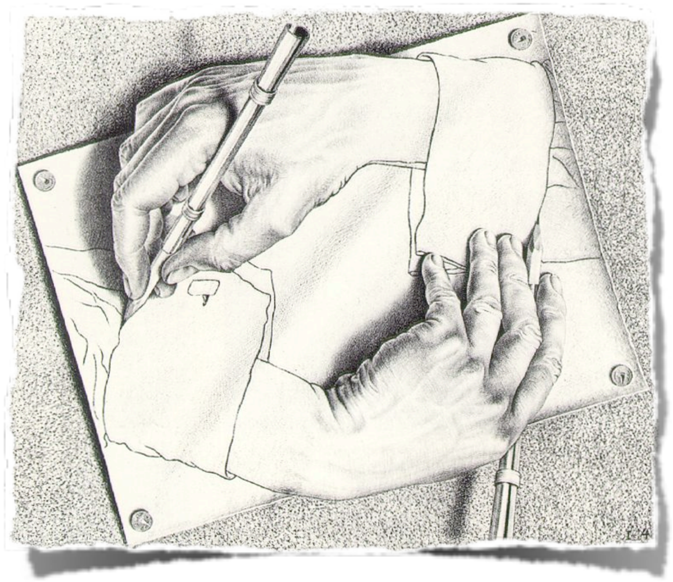
Summary of L_2 Result

- *Thm:* $(1+\epsilon)$-factor approx. (w/p $1-\delta$) in $\tilde{O}(\epsilon^{-2} \ln \delta^{-1})$ space.
- *Proof Ideas:*
  1) *First attempt:* Use AMS technique.
  2) *Road block:* Can't sketch product distribution.
  3) *Bilinear sketch:* Product of sketches was sketch of product!
  4) *PANIC:* No longer 4-wise independence.
  5) *Relax:* We didn't need full 4-wise independence.

a) Neat Result for L_2

***b)* Sketching Sketches**

c) Other Results



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- *N.B.* If *median* were *mean* we'd have a dimensionality reduction result that doesn't exist. [Brinkman, Charikar '03]

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- To sketch product distribution need $z = yM_x$

$$z = \underbrace{(\quad y \quad)}_{n} \underbrace{\begin{pmatrix} (\quad x \quad) & 0 & \ldots & 0 \\ 0 & (\quad x \quad) & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & (\quad x \quad) \end{pmatrix}}_{n^2}$$

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- *The Problem:*

  Need to take median of multiple inner sketches before taking outer sketch.

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$$z = \underbrace{\left(\quad y \quad \right)}_{n} \underbrace{\left(\qquad\qquad M_x \qquad\qquad \right)}_{n^2}$$

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$$\begin{array}{ccc} \mathbb{R}^{n^2} & \longmapsto & \mathbb{R}^n \\ a & \longrightarrow & M_x a \end{array} \qquad \begin{array}{ccc} \mathbb{R}^n & \longmapsto & \mathbb{R} \\ M_x a & \longrightarrow & yM_x a \end{array}$$

- *The Problem:*

  Need to take median of multiple inner sketches before taking outer sketch.

  The size of the inner sketch is large.

L_I Result

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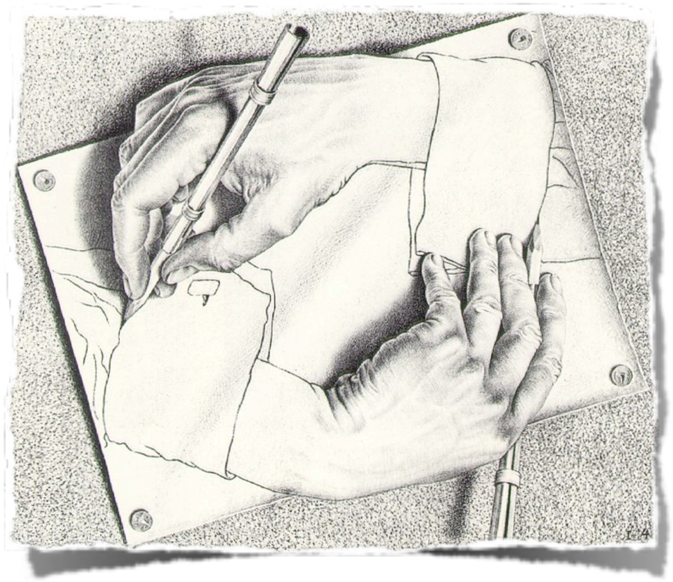
Repeat Õ(ln δ^{-1}) times and take median.

a) Neat Result for L_2

b) Sketching Sketches

c) Other Results



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  Can't $(1+\epsilon)$-factor approximate in $o(n)$ space

  Can $\pm\epsilon$ using algorithms for approx. entropy.

[Chakrabarti, Cormode, McGregor '07]

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- ***Distributed Model:***

  Player 1 sees $(3,\cdot), (5,\cdot), (2,\cdot), (3,\cdot), (7,\cdot), (1,\cdot), (3,\cdot), (6,\cdot), \ldots$

  Player 2 sees $(\cdot,5), (\cdot,3), (\cdot,7), (\cdot,4), (\cdot,1), (\cdot,2), (\cdot,9), (\cdot,6), \ldots$

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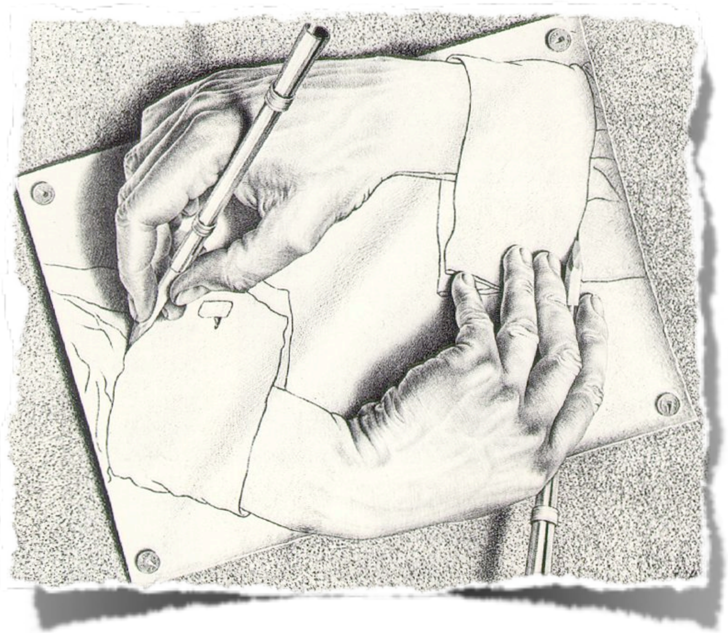
- *Additive Approximation for $L_1(s-r)$:*

$$L_1(p-q) = \sum_i p_i L_1(q - q^i)$$

  where q^i is q conditioned on first term equals i.

  [Guha, McGregor, Venkatasubramanian '06]



Main Results

Can estimate $L_2(r-s)$ well using neat extension of AMS sketch.

Can estimate $L_1(r-s)$ up to $O(\log n)$ factor using p-stable distributions.

Can estimate mutual information additively using entropy algorithms.

Questions?