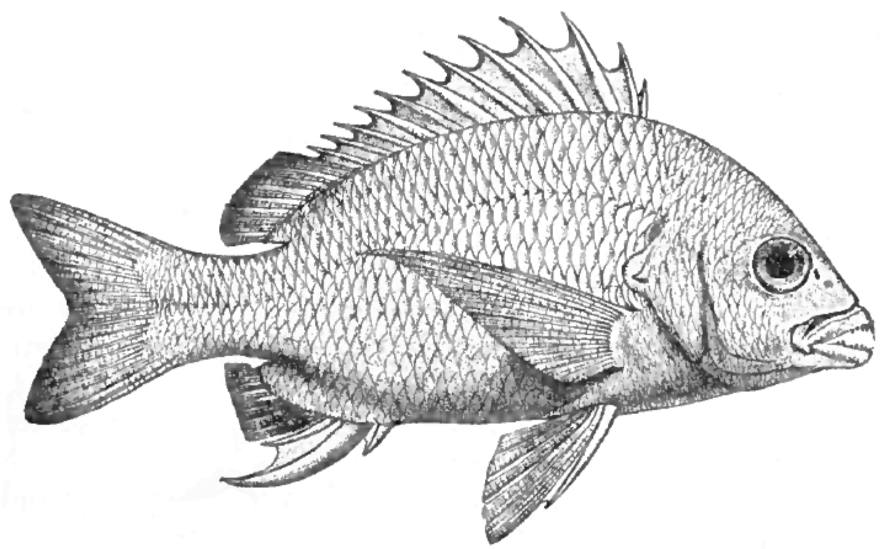
Lower Bounds for Quantile Estimation in Random-Order and Multi-Pass Streams

Sudipto Guha (UPenn)   Andrew McGregor (UCSD)



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- *Stream:* m elements from a universe of size n:

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- *Previous work:* quantiles, frequency moments, histograms, clustering, entropy, graph problems...

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- What about the *random-order model (ROM)*?
  - Form of average case analysis
  - Stream of independent samples
  - Uncorrelated fields in a database...
- *Previous Work:*
  - Frequent elements [Demaine, Lopez-Ortiz, Munro '02]
  - Entropy & Distances [Guha, McGregor, Venkatasubramanian '06]
  - Histograms [Guha, McGregor '07]
  - Quantiles... [Munro, Paterson '78], [Guha, McGregor '06]

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- *Main Questions:*
  Are these ROM results possible in the AOM model?
  Can these ROM results be improved?

Results

- *Thm:* For a stream in *random order*:
  a) 1-pass, O(polylg m)-space, $\tilde{O}(m^{1/2})$-approx
  b) O(lg lg m)-pass, O(polylg m)-space exact selection

- *Thm:* For a stream in *adversarial order*:
  a) 1-pass, $\tilde{O}(m^{1/2})$-approx requires $\Omega(m^{1/2})$ space
  b) O(polylg m)-space exact requires $\Omega(\lg m)$ passes

- *Bonus Thm:* For a stream in *random order*, a single pass, t-approx requires $\Omega(m^{1/2} t^{-3/2})$ space.

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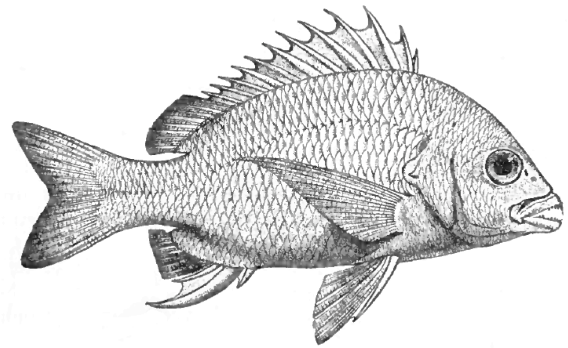
2: Lower-Bound *(Random)*

3: Lower-Bound *(Advesarial)*



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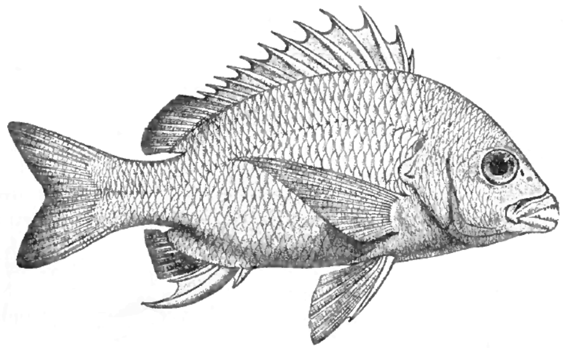

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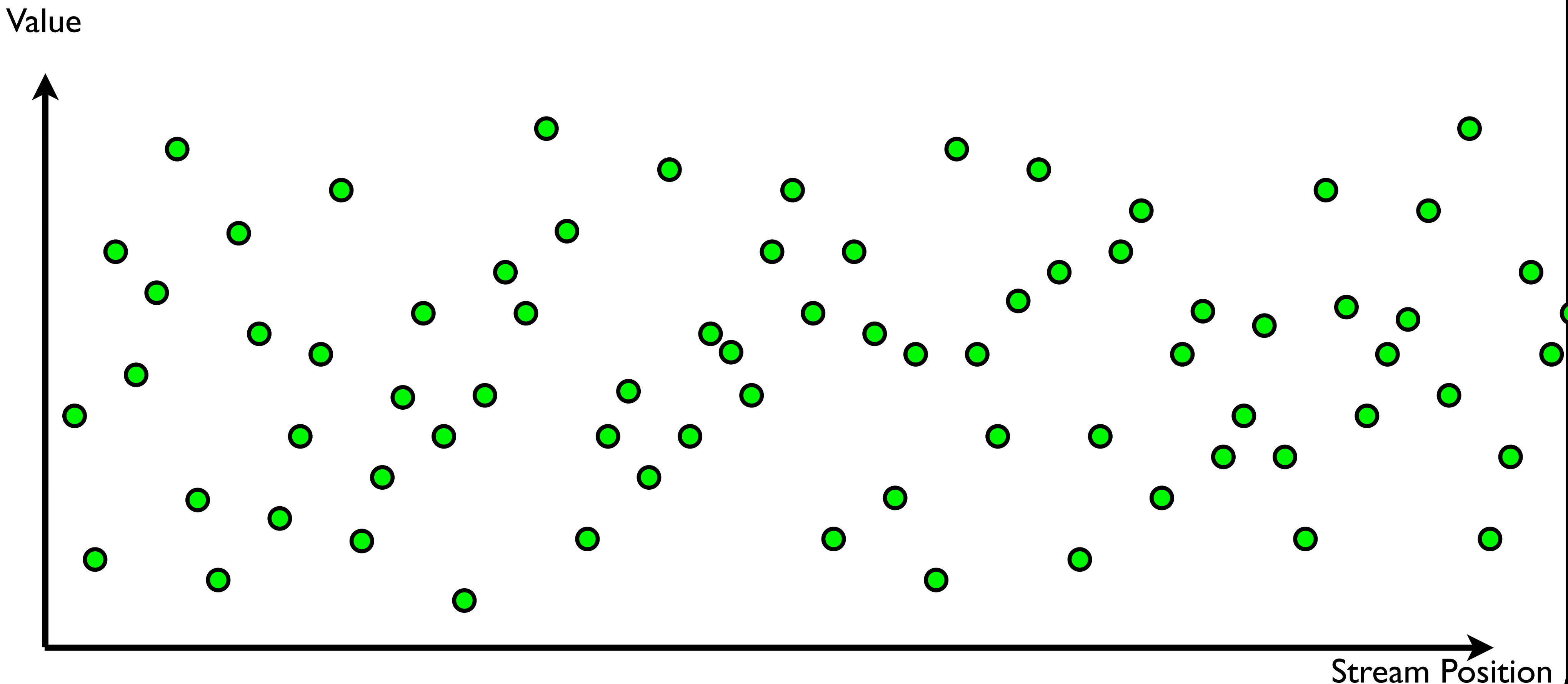

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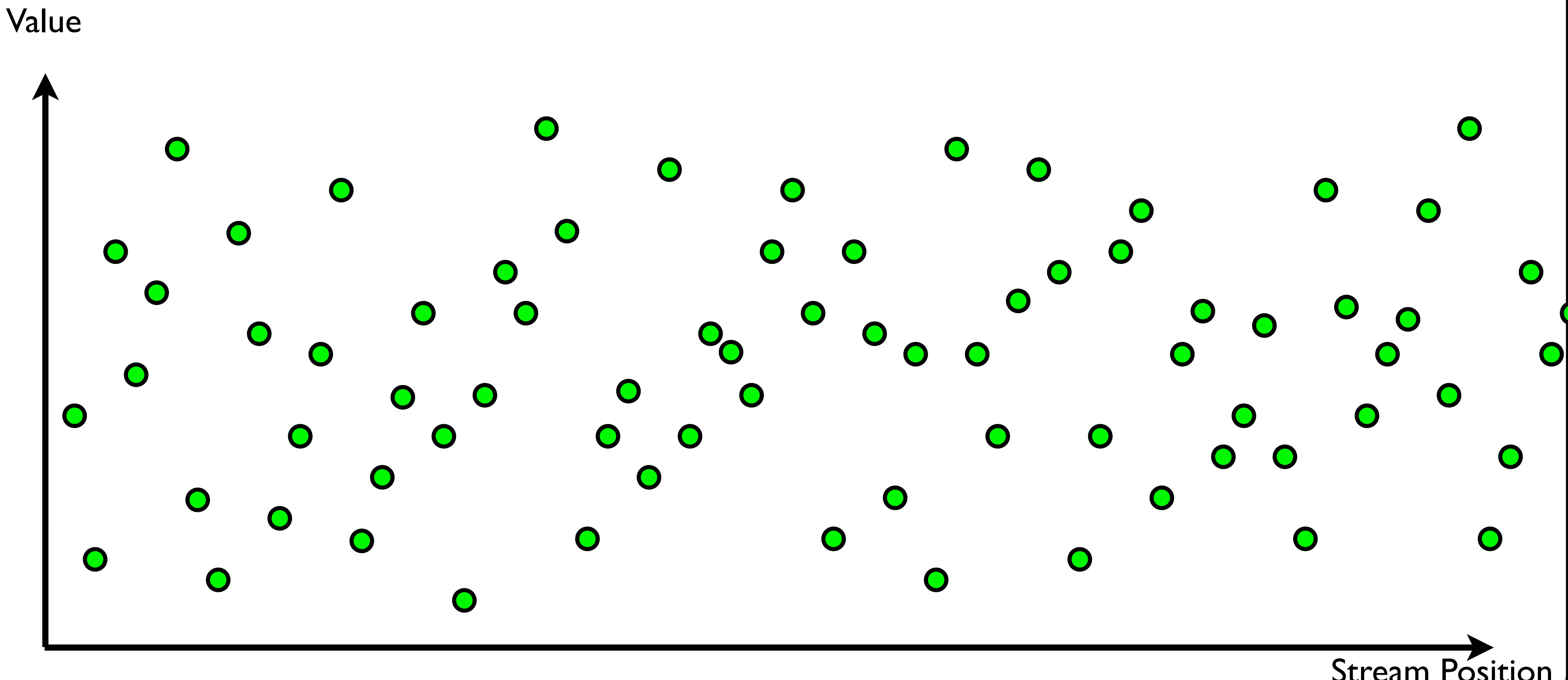

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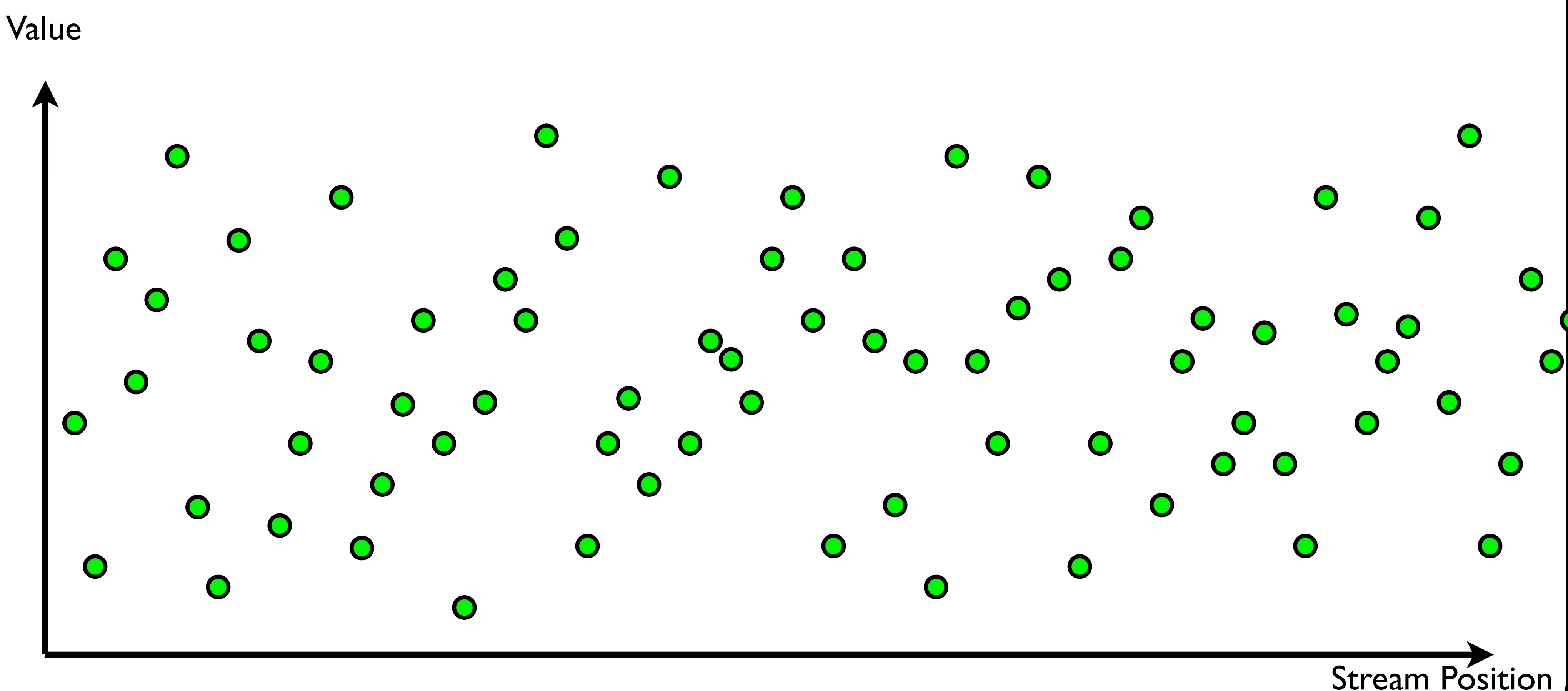

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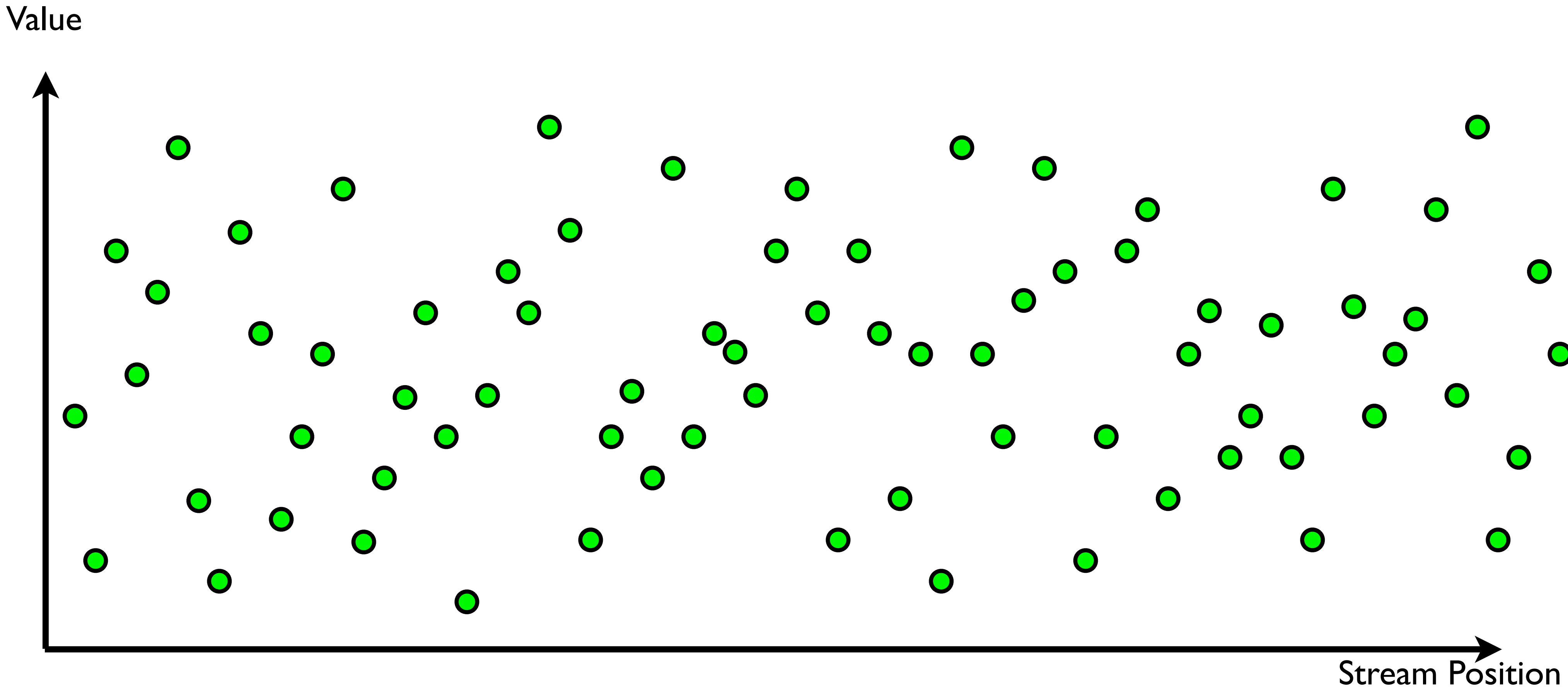

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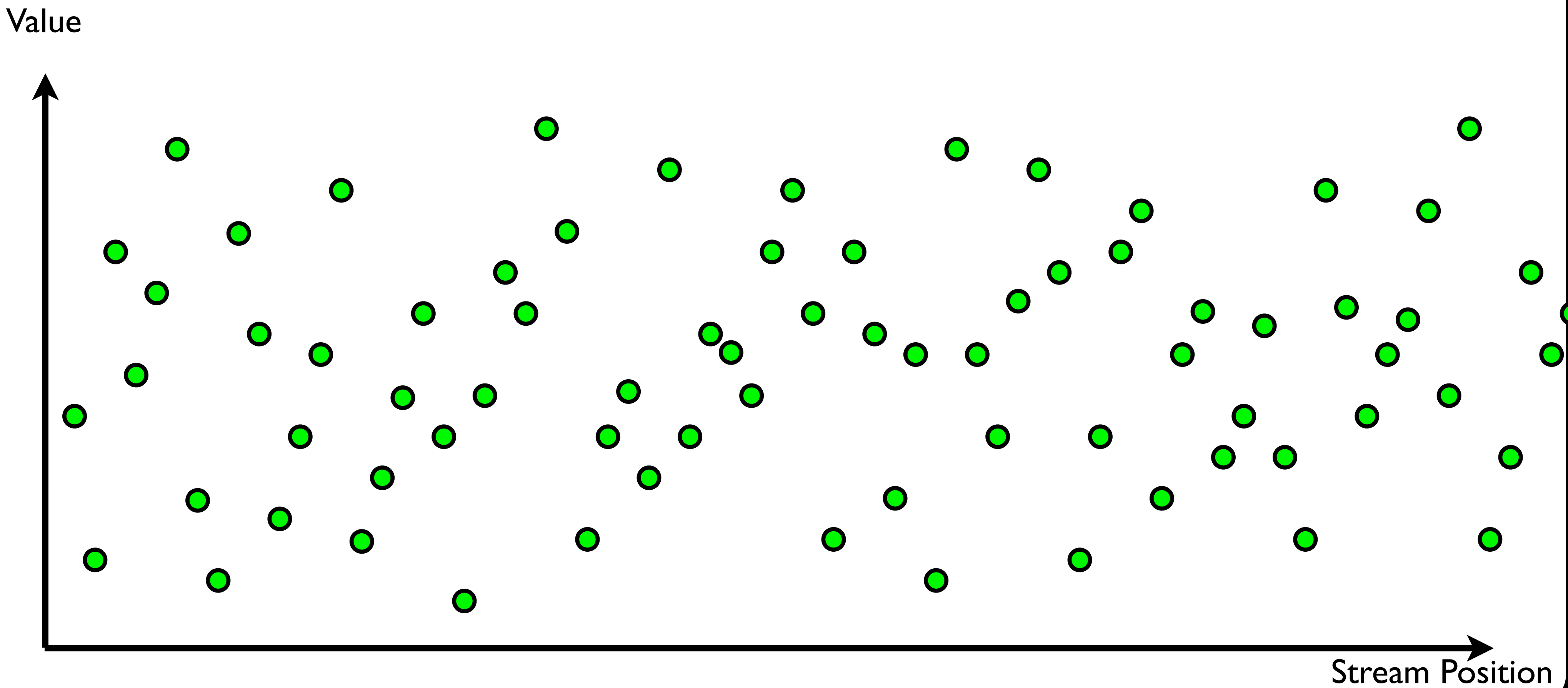

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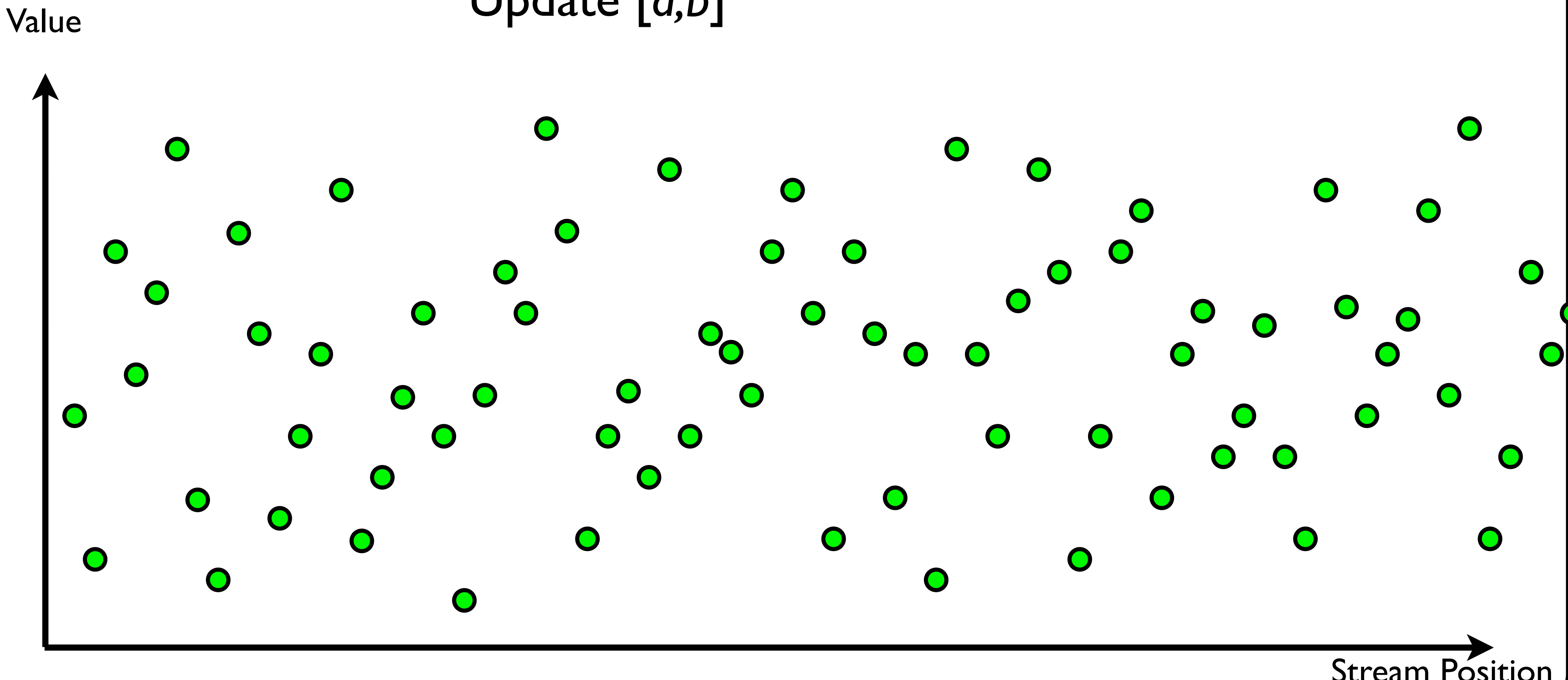

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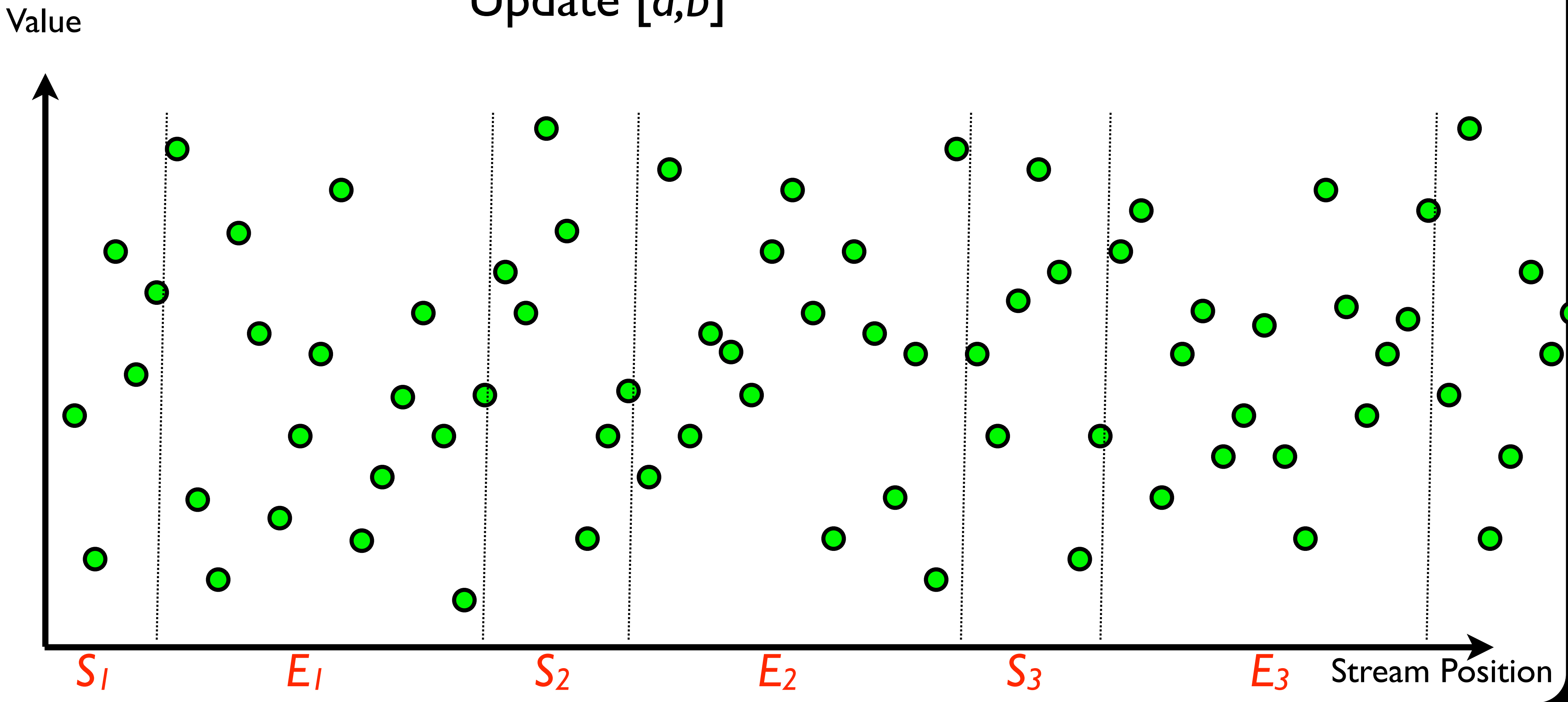

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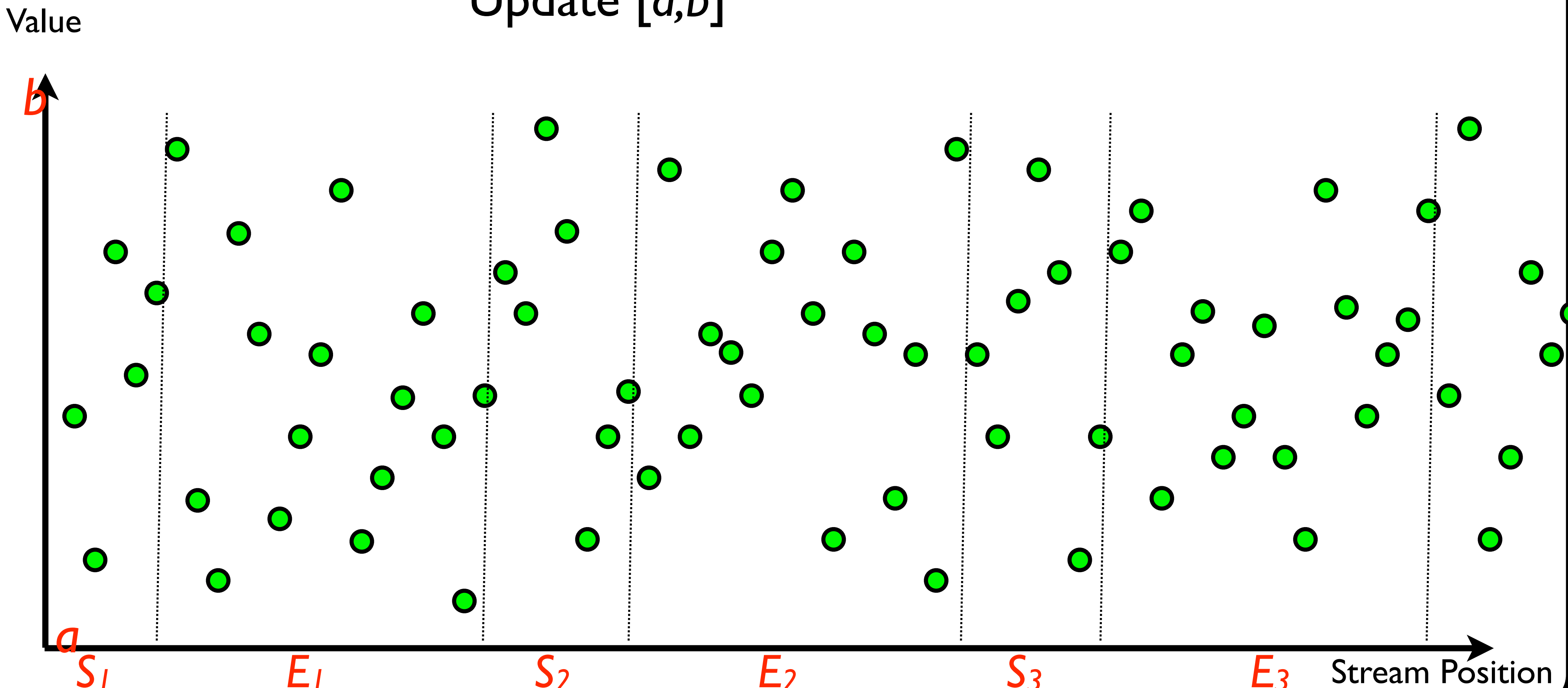

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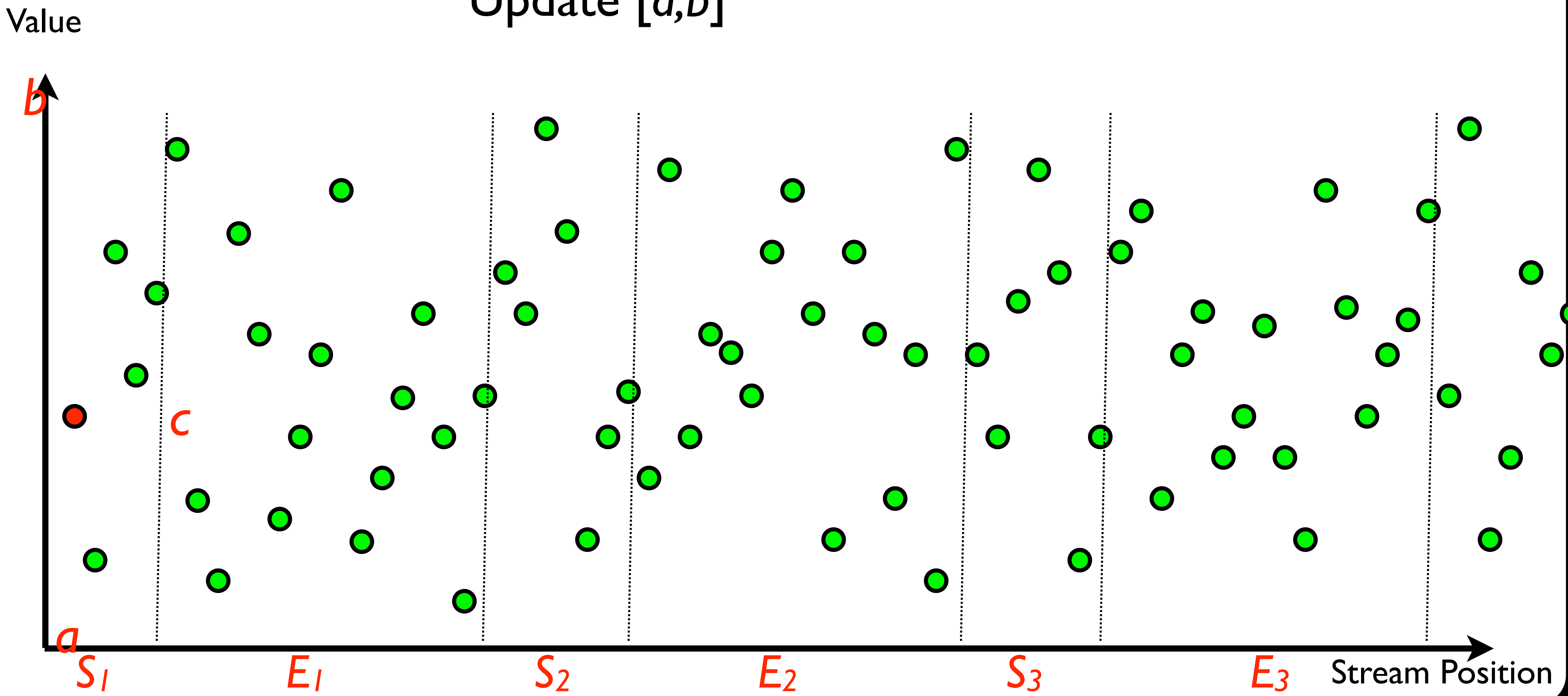

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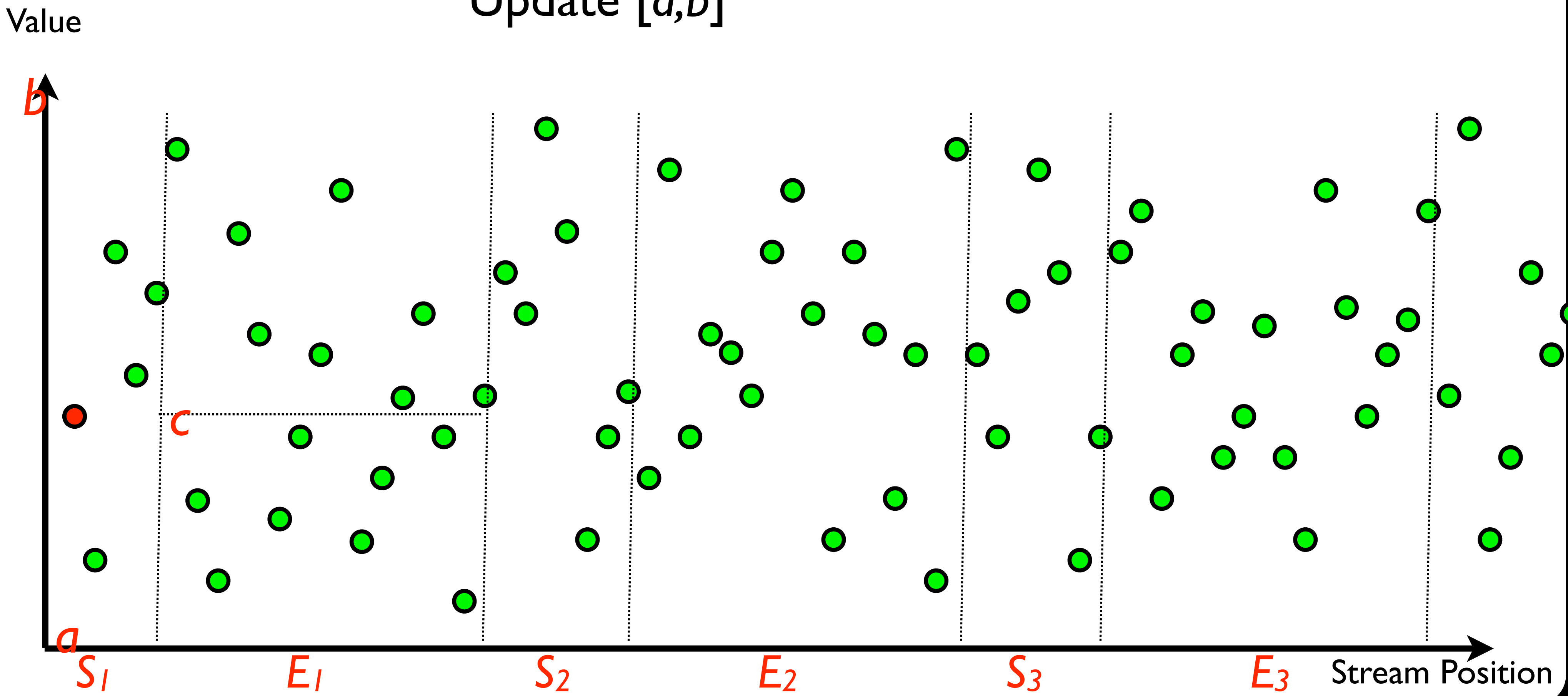

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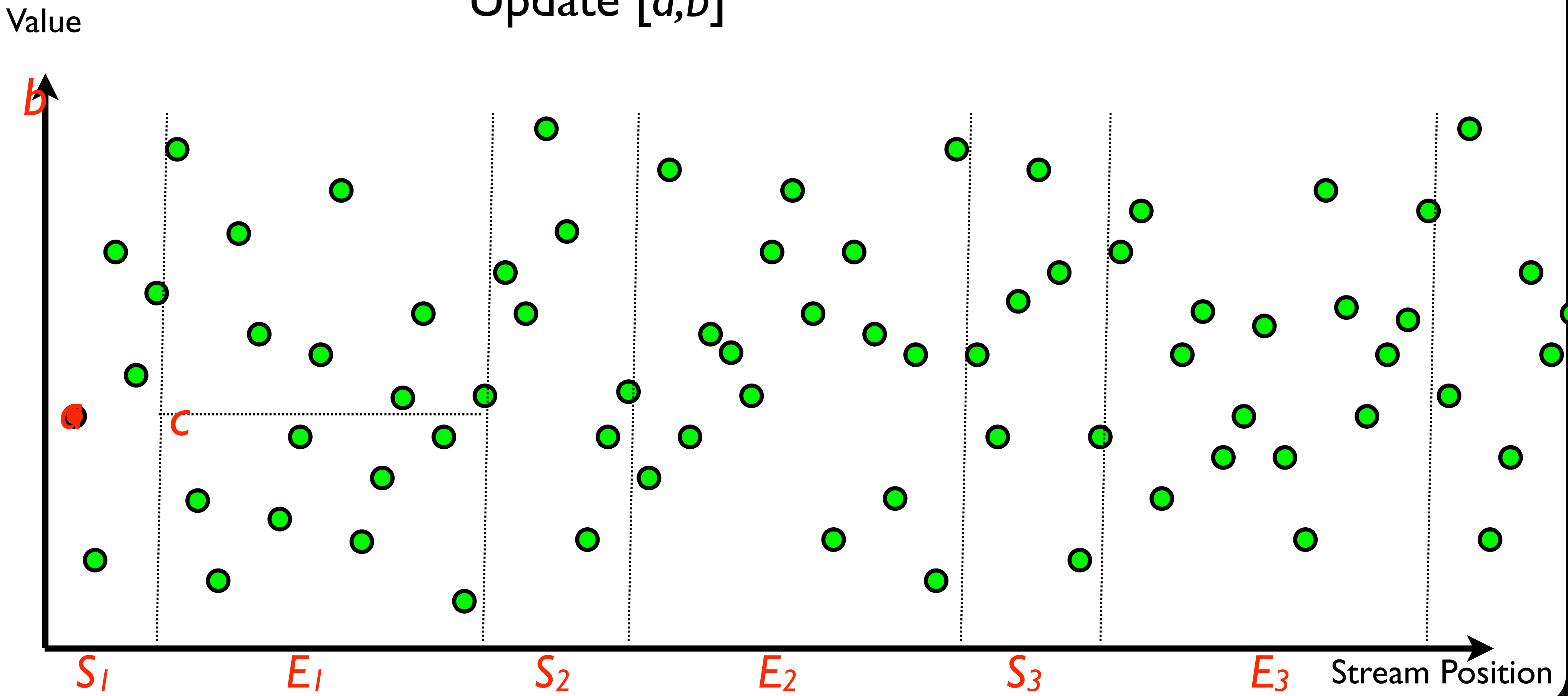

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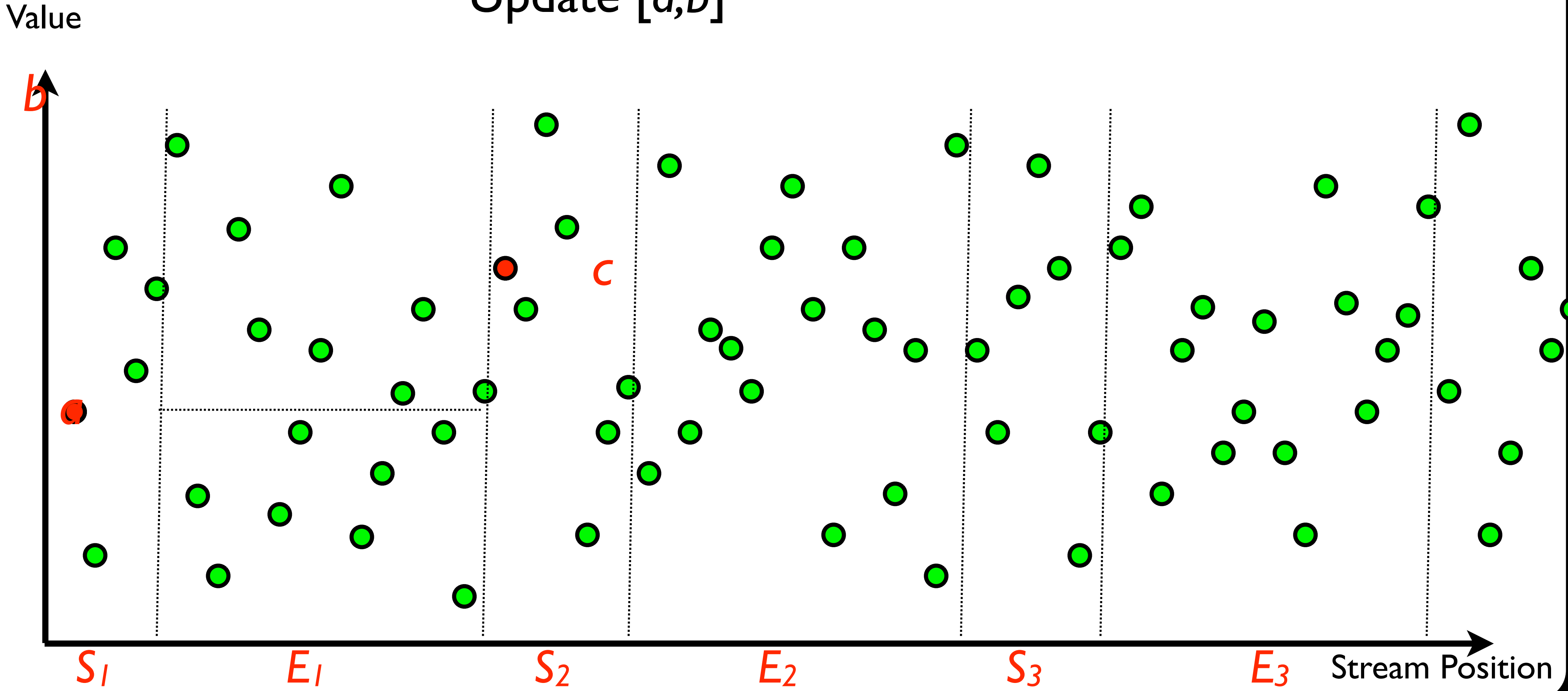

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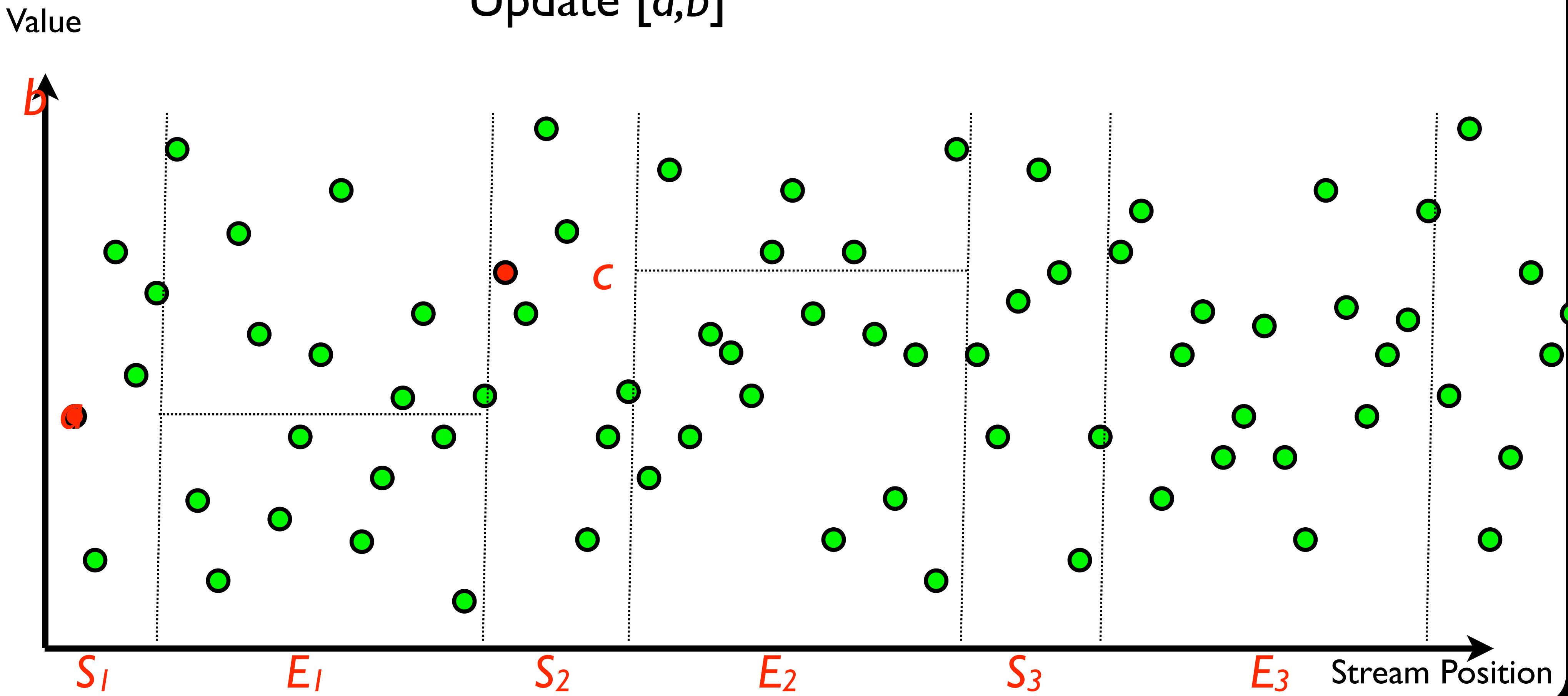

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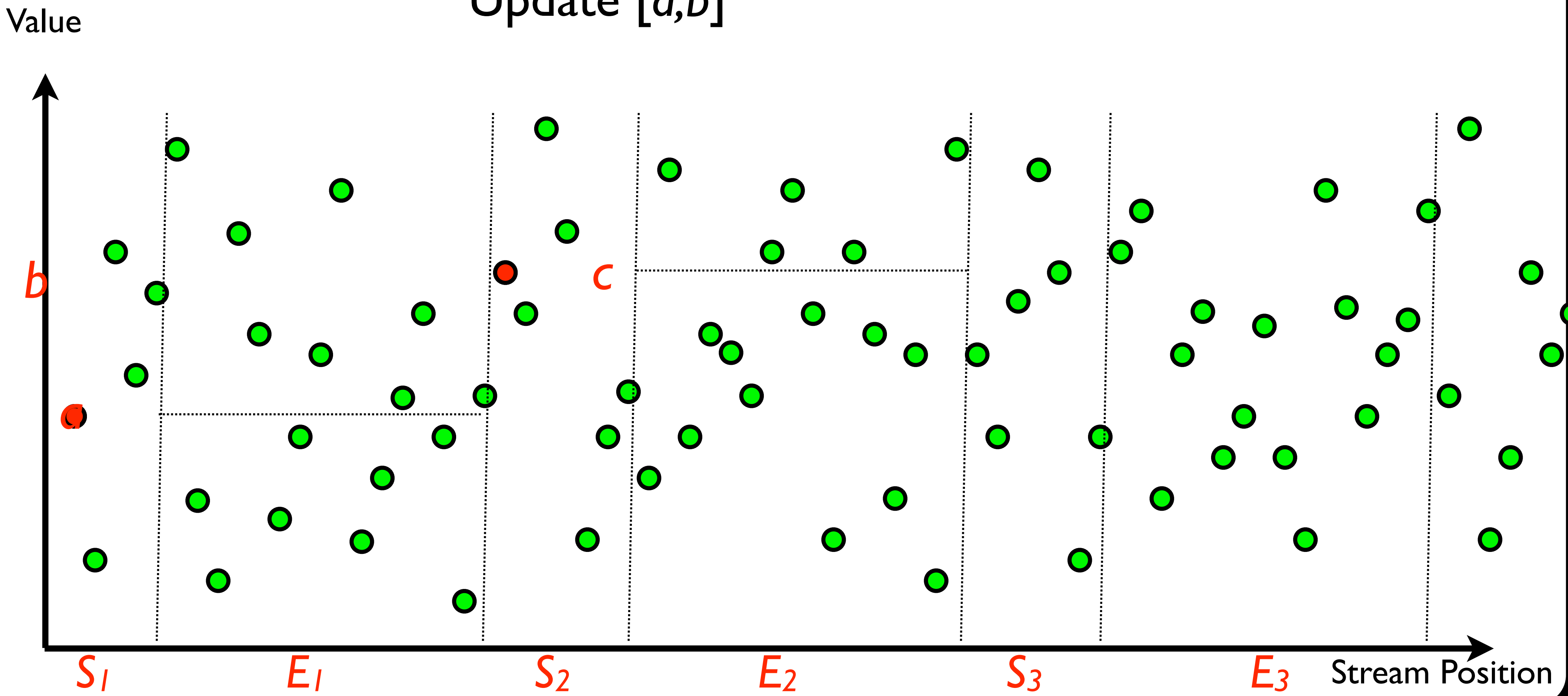

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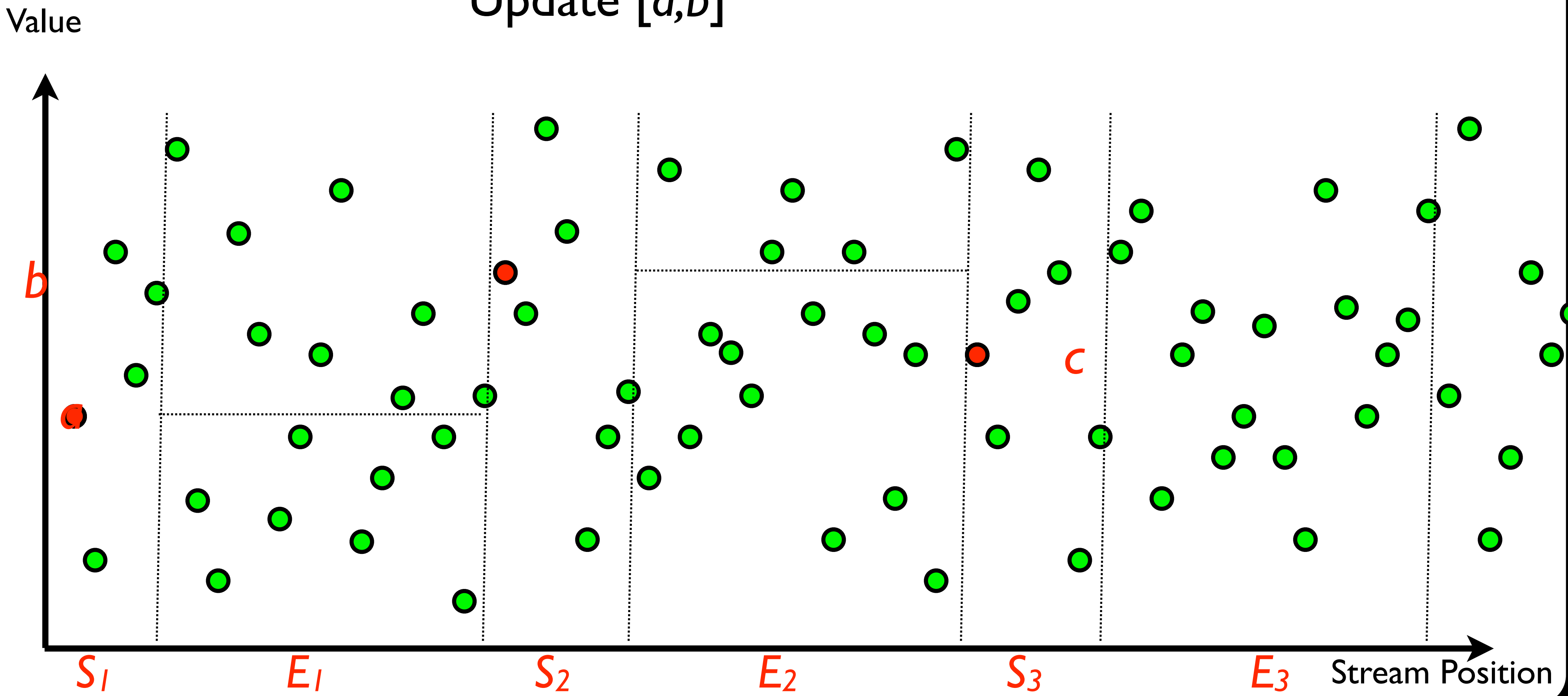

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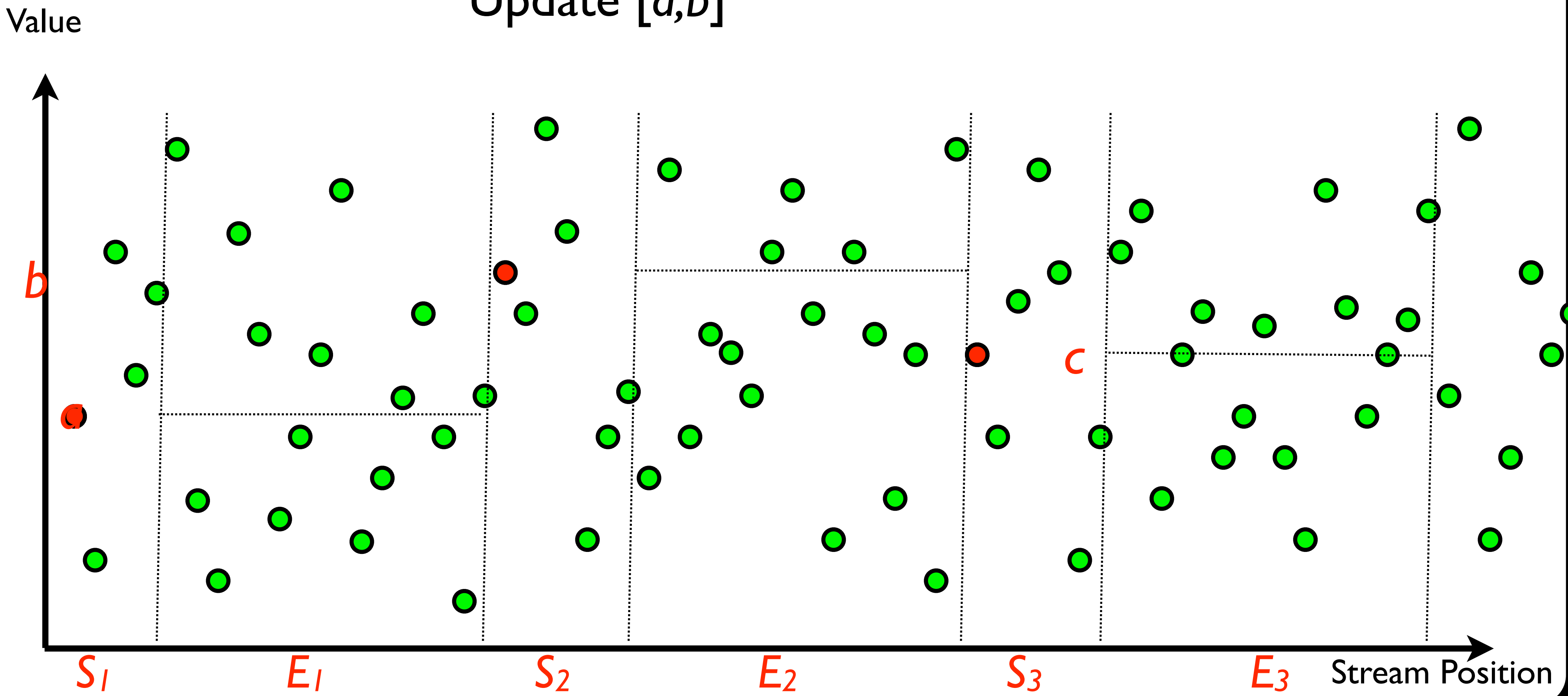

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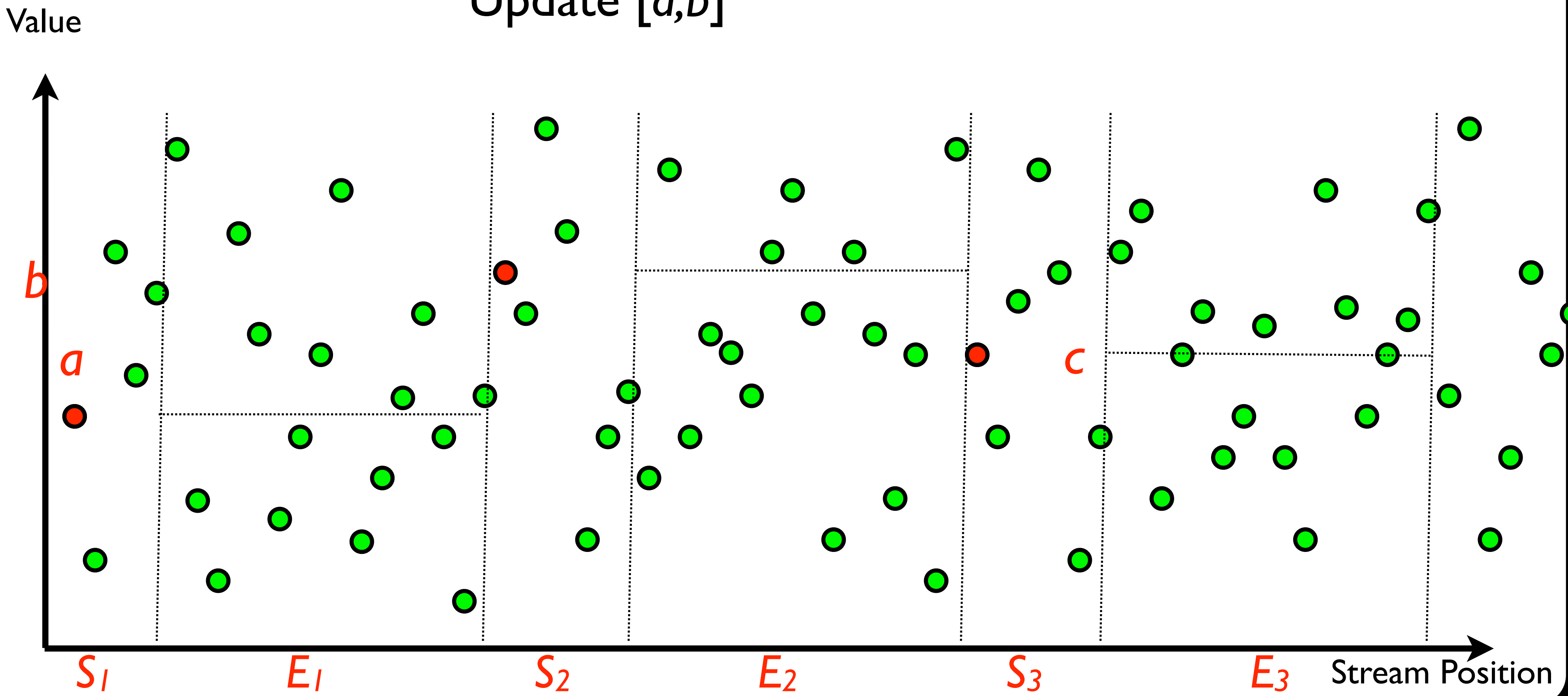

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- *Lemma:* Expect rank(b)-rank(a) to half per-phase, hence $p = O(\lg m)$ w.h.p.
- *Thm:* If stream order is random, we return an element with rank $m/2 \pm t$ using O(log m) space.

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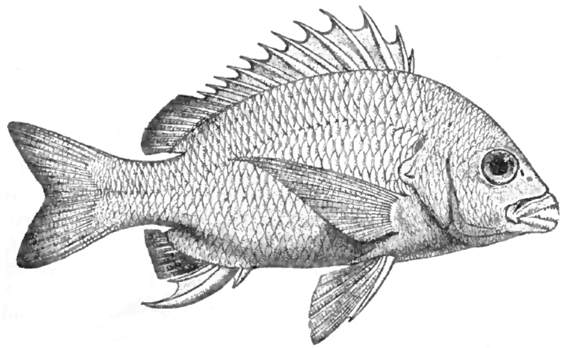

Alice

length *m*

binary string *x*

Bob

index *i* in

range [*m*]

INDEX: "What's the value of x_i?"

Requires $\Omega(n)$ bits transmitted.

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$2+x_1$ ... $2i+x_i$ ... $2m+x_m$

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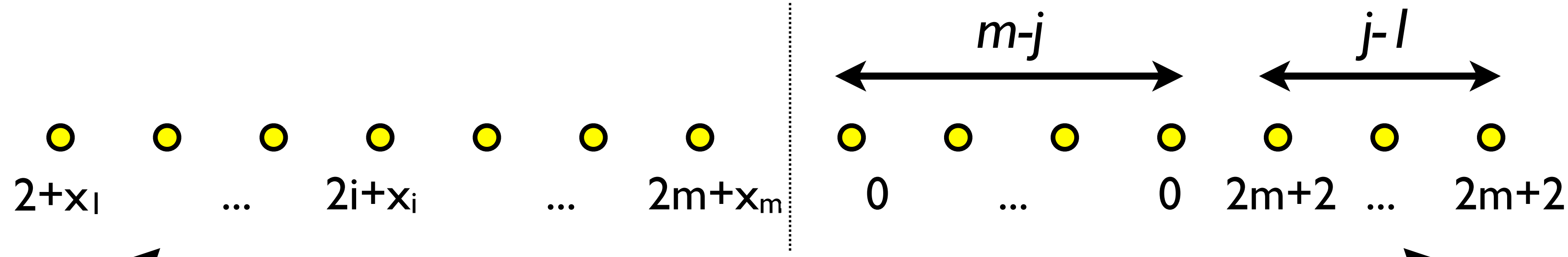

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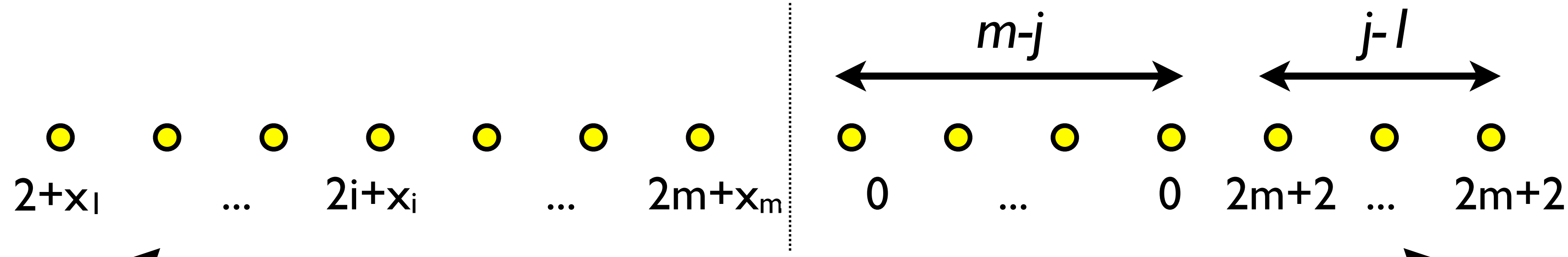

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m-j j-l

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Alice
length m
binary string x

MEMORY STATE OF ALGORITHM

Bob
index i in range $[m]$

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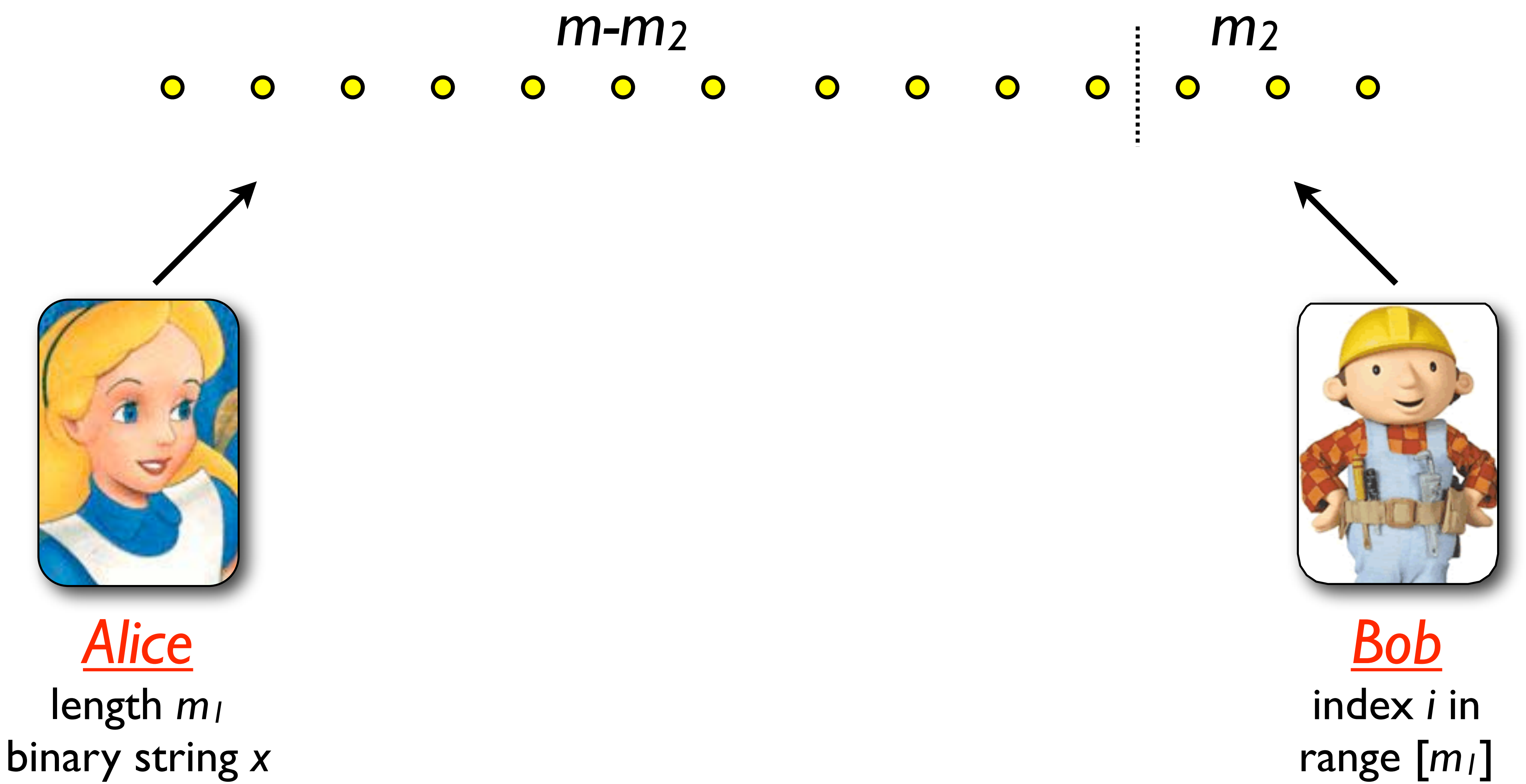
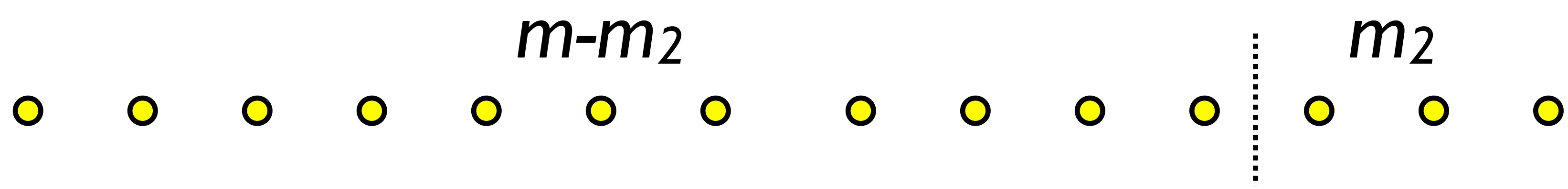
- *Thm:* $S=\Omega(m)$ [Henzinger, Raghavan, Rajagopalan '99]



m-m2
m2
Alice
length m1
binary string x
Bob
index i in
range [m1]



Alice: picks b randomly from $[m_1]$ and inserts a random permutation of,

$$\{\underbrace{0, \ldots, 0}_{(m-m_1-m_2-b)/2}, 2+x_1, \ldots, 2m_1+x_{m_1}, \underbrace{2m+2, \ldots, 2m+2}_{(m-m_1-m_2+b)/2}\}$$

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length m_1

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m-m_2 m_2

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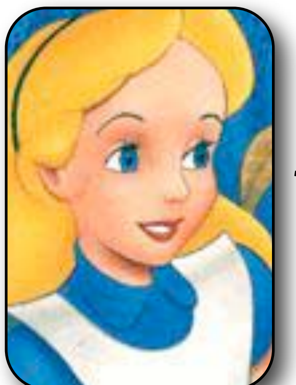
MEMORY STATE OF ALGORITHM and "b"



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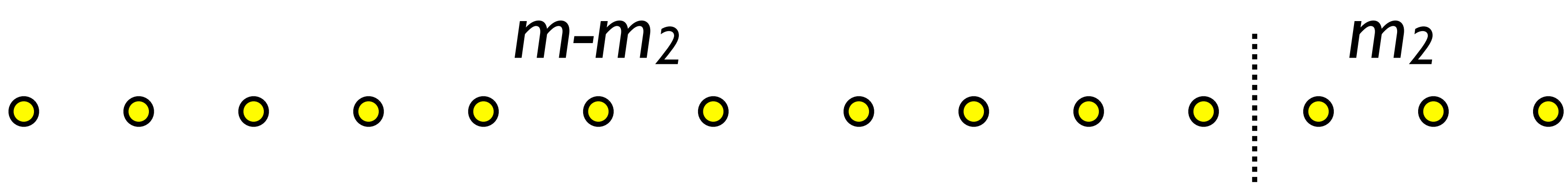

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MEMORY STATE OF ALGORITHM and "b"

Bob: inserts a random permutation of,

$$\{\underbrace{0, \quad \dots \quad, 0}_{(m_2+b-j-1)/2}, \underbrace{2m+2, \dots, 2m+2}_{(m_2-b+j)/2}\}$$

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length m_1
binary string x

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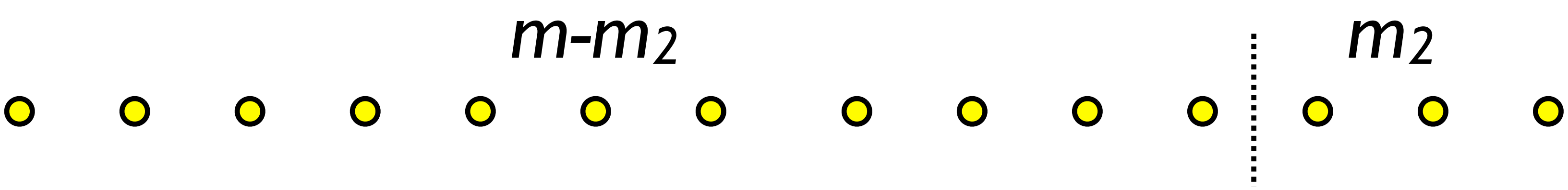

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Alice

length m_1

binary string x

MEMORY STATE OF ALGORITHM and "b"

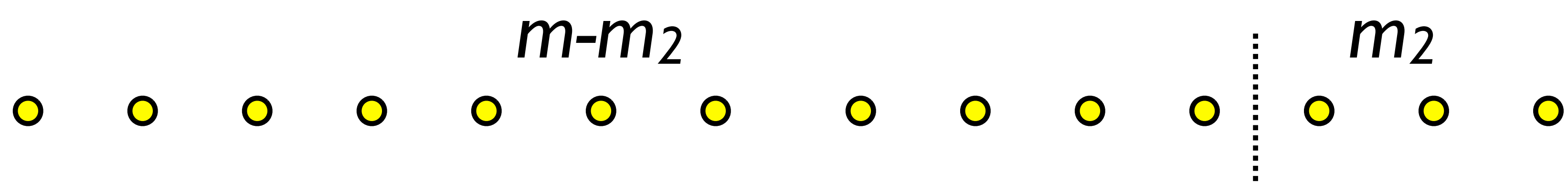
Bob: inserts a random permutation of,

$$\{\underbrace{0, \quad \ldots \quad, 0}_{(m_2+b-j-1)/2}, \underbrace{2m+2, \ldots, 2m+2}_{(m_2-b+j)/2}\}$$

Bob

index i in range $[m_1]$

- If $m_1=o(\sqrt{m_2})$ and $m_1 m_2=o(m)$ then ordering is "close" to random in L_1 sense.



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m-m_2 | m_2

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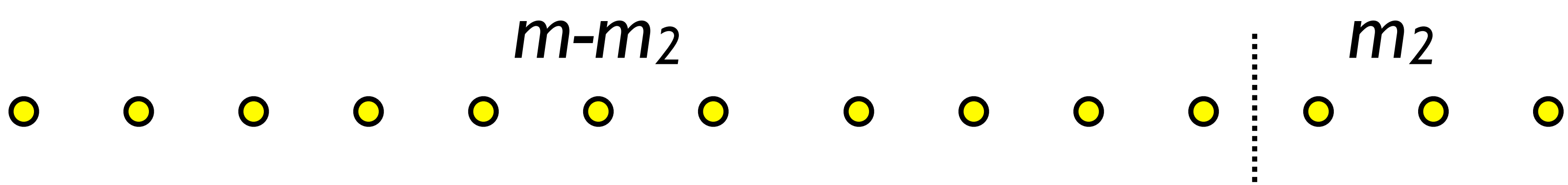

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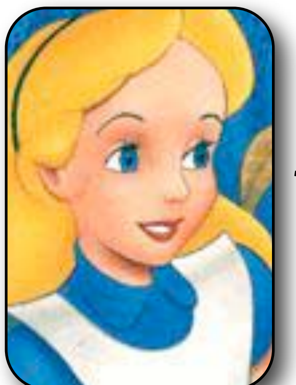

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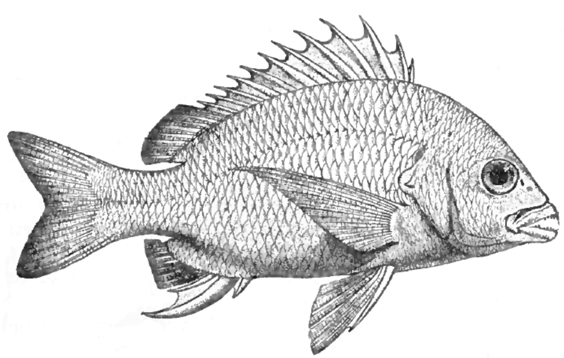
1: Algorithm *(Random)*

2: Lower-Bound *(Random)*

3: Lower-Bound *(Advesarial)*



Pointer Chasing

Alice
function
f_A: [t]→[t]

Bob
function
f_B: [t]→[t]

Pointer Chasing

Alice

function

$f_A: [t] \to [t]$

Compute

$$f_A(f_B(\ \dots\ (f_A(f_B(f_A(1))))\ \dots\ ))$$

$\longleftarrow$ k $\longrightarrow$

Bob

function

$f_B: [t] \to [t]$

Pointer Chasing

Alice

function

$f_A: [t] \to [t]$

Compute

$$f_A(f_B(\ldots (f_A(f_B(f_A(1)))) \ldots))$$

$\longleftrightarrow$ k

If Bob speaks first:

k messages: $O(k \log t)$ bits.

k-1 messages: $\Omega(t)$ bits.

[Nisan, Wigderson '93]

Bob

function

$f_B: [t] \to [t]$

Multi-Party Pointer Chasing

Alice
function
$f_A: [t] \to [t]$

Bob
function
$f_B: [t] \to [t]$

Claude
function
$f_C: [t] \to [t]$

...

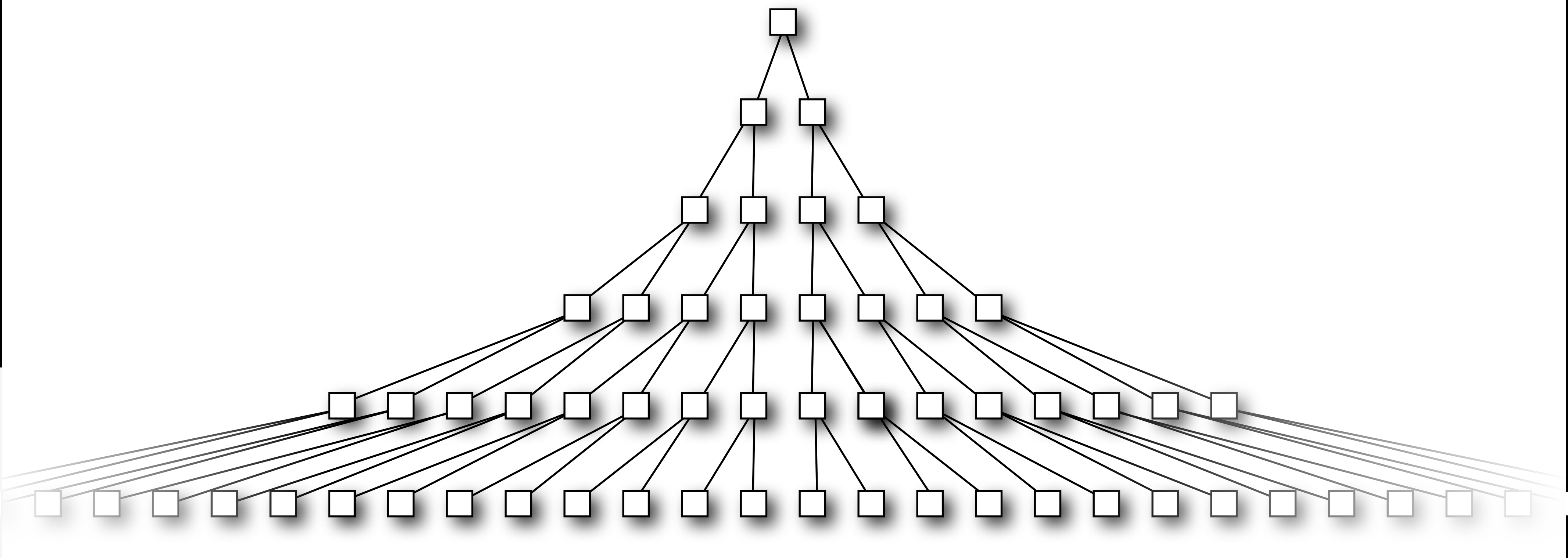
Zebedee
function
$f_Z: [t] \to [t]$

Compute $f_Z(\ \dots f_C(f_B(f_A(1))) \dots)$

Order of speaking "Z, ..., C, B, A"

- k rounds: $O(k \log t)$ bits.
- k-1 rounds: $\Omega(t)$ bits.

Proof Sketch (k=3)



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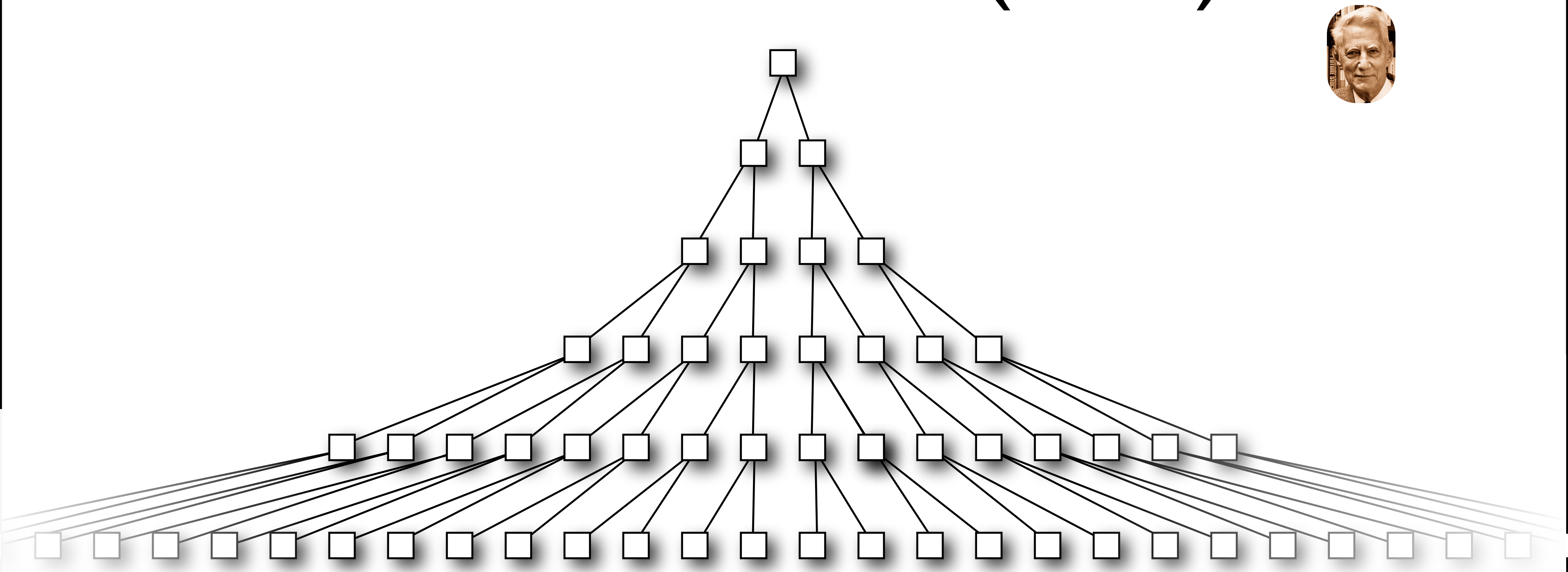

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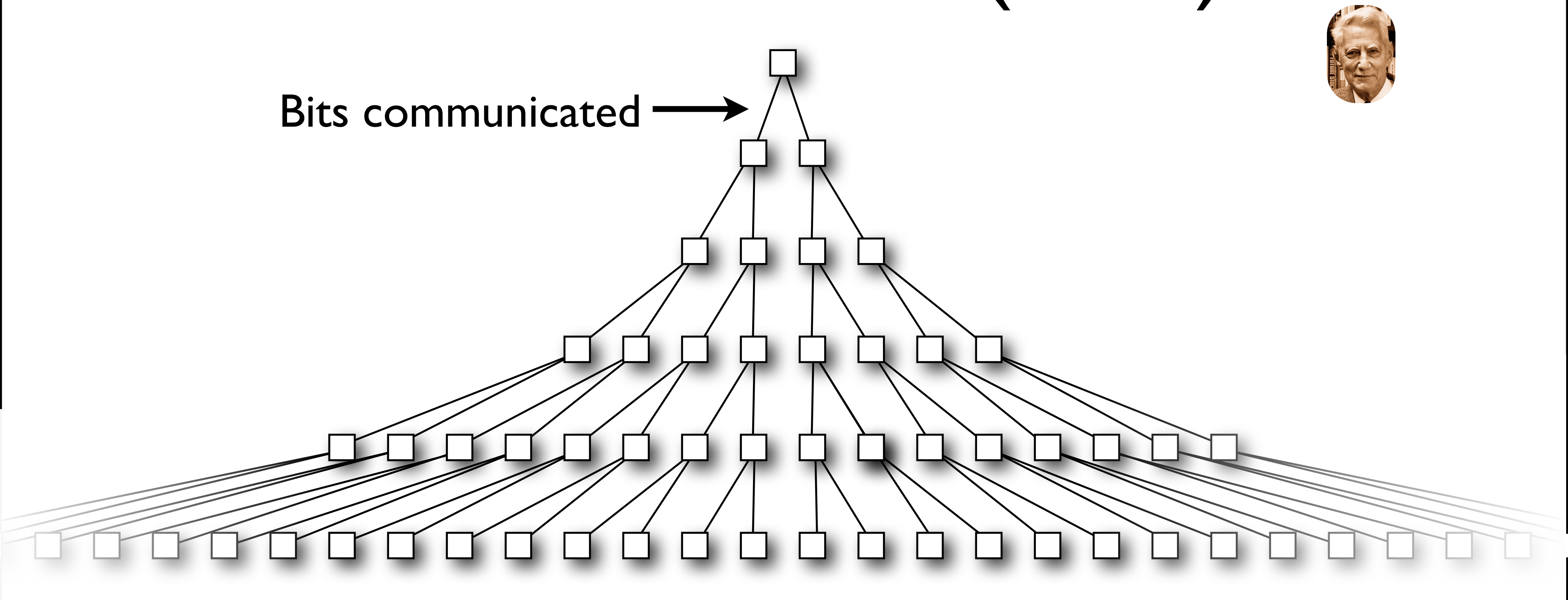

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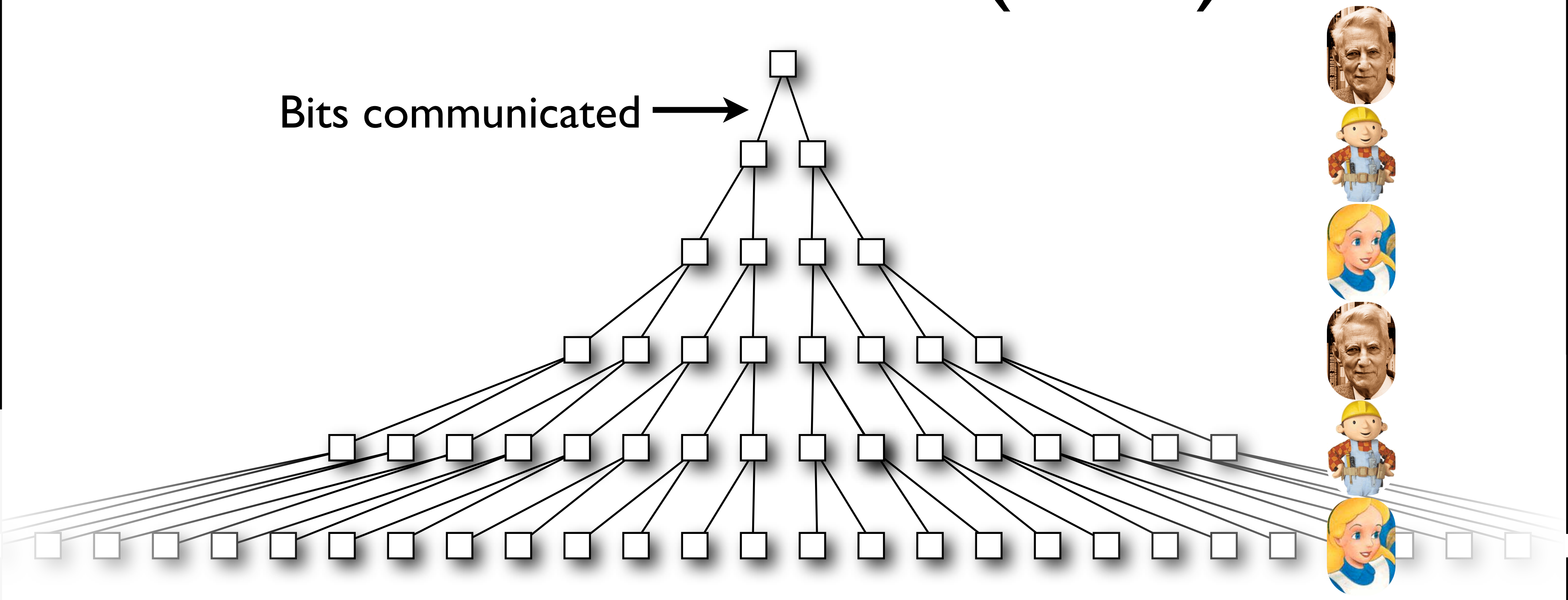
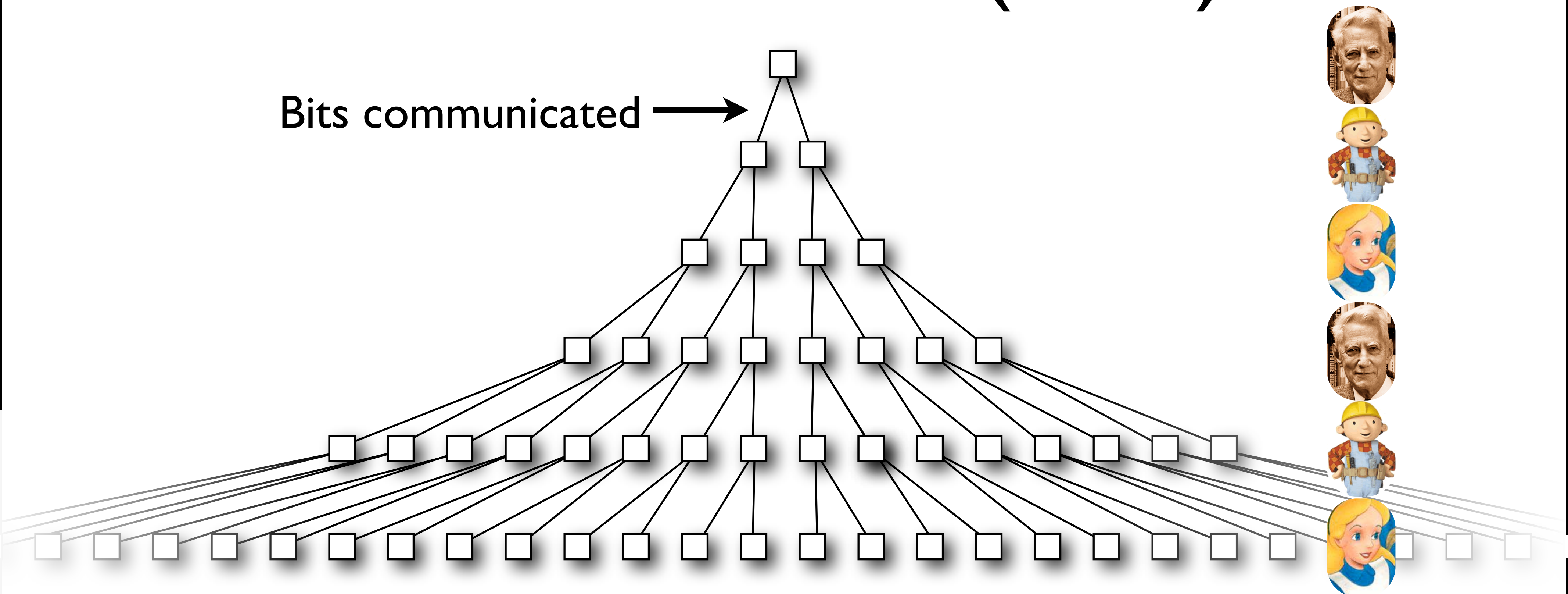
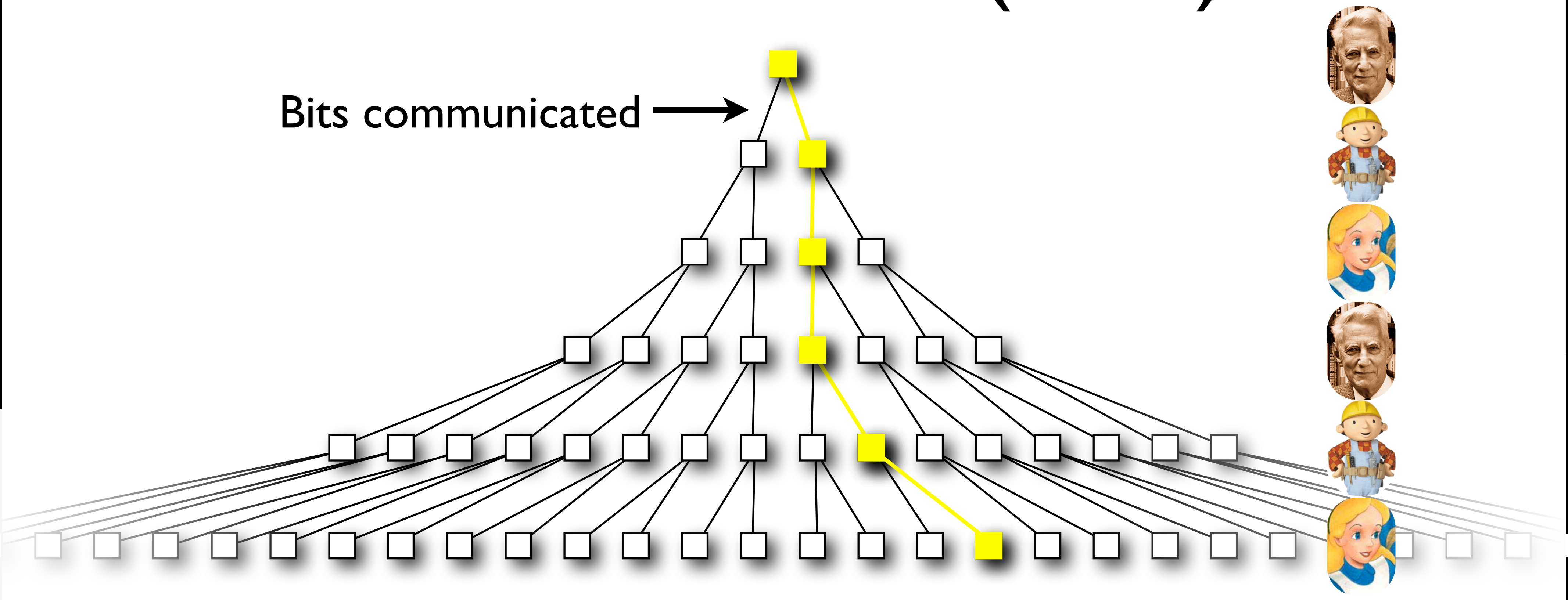


Proof Sketch (*k*=3)



- Consider *deterministic* protocols and *random* f_A, f_B, f_C

Proof Sketch (k=3)



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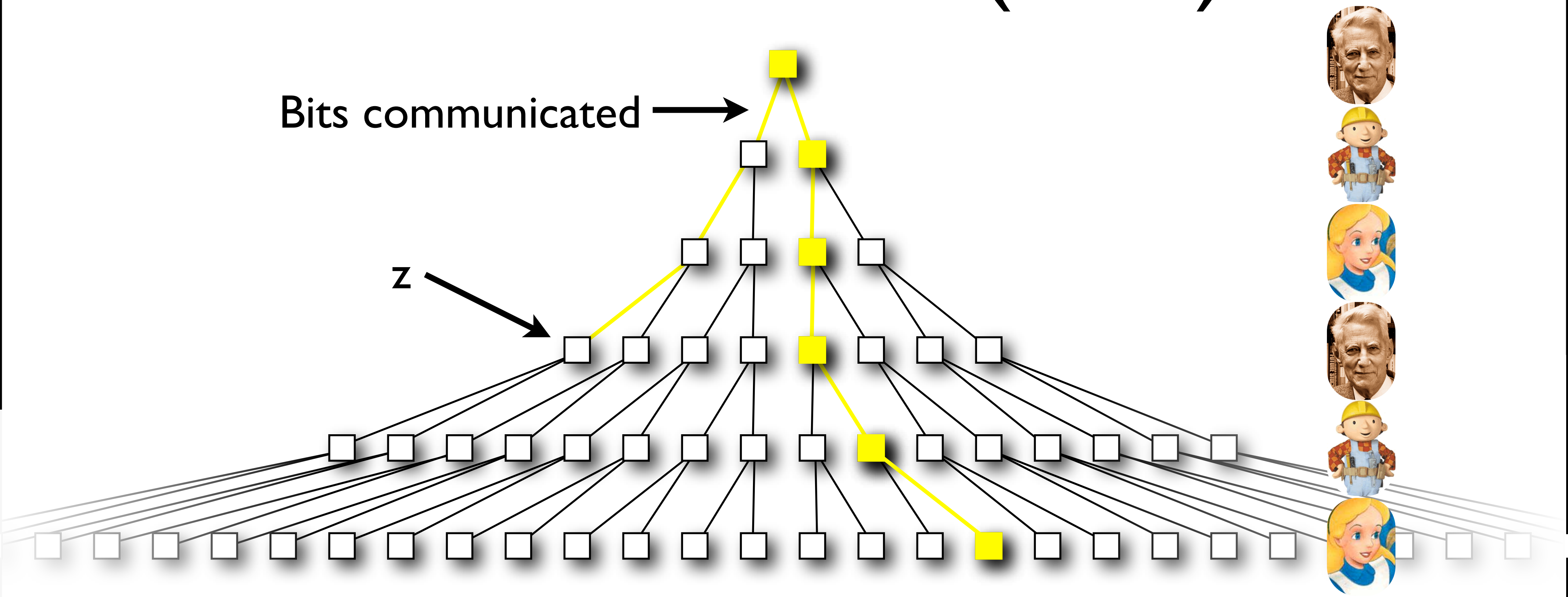

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- Each node *z* defines random variables $z(f_A)$, $z(f_B)$, $z(f_C)$

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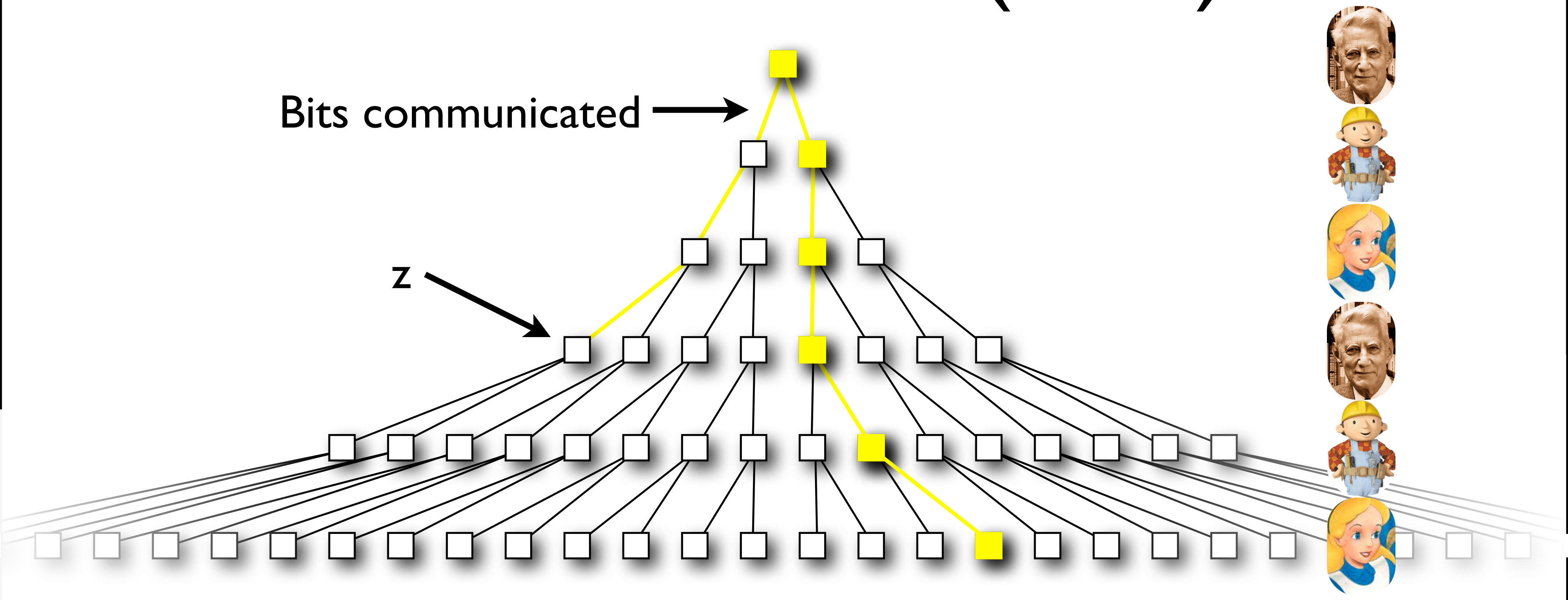
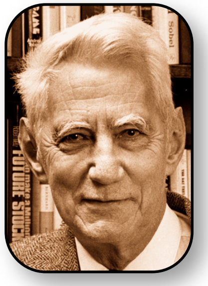


- Consider *deterministic* protocols and *random* f_A, f_B, f_C
- Each node *z* defines random variables $z(f_A)$, $z(f_B)$, $z(f_C)$
- Induction: For each *z*, entropy of variables is high.

Reduction to Selection

f_A: $[t] \to [t]$

f_B: $[t] \to [t]$

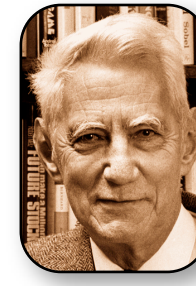


f_C: $[t] \to [t]$

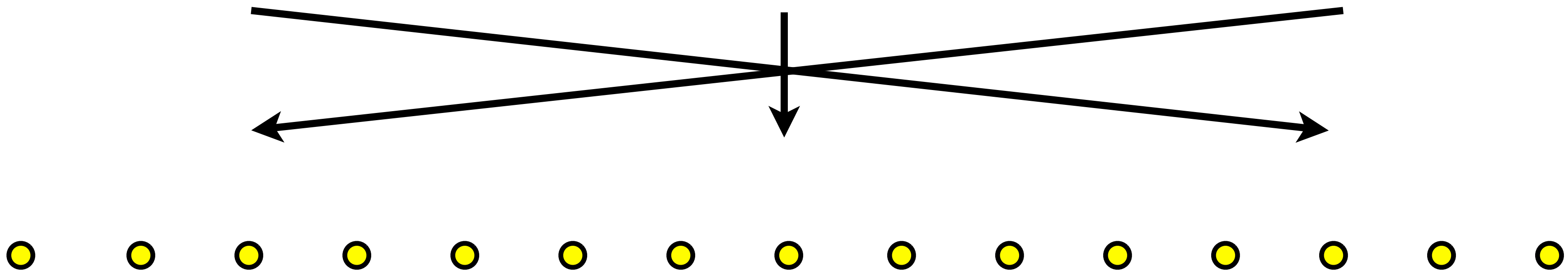
Reduction to Selection

f_A: $[t] \rightarrow [t]$

f_B: $[t] \rightarrow [t]$



f_C: $[t] \rightarrow [t]$



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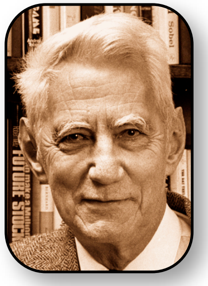

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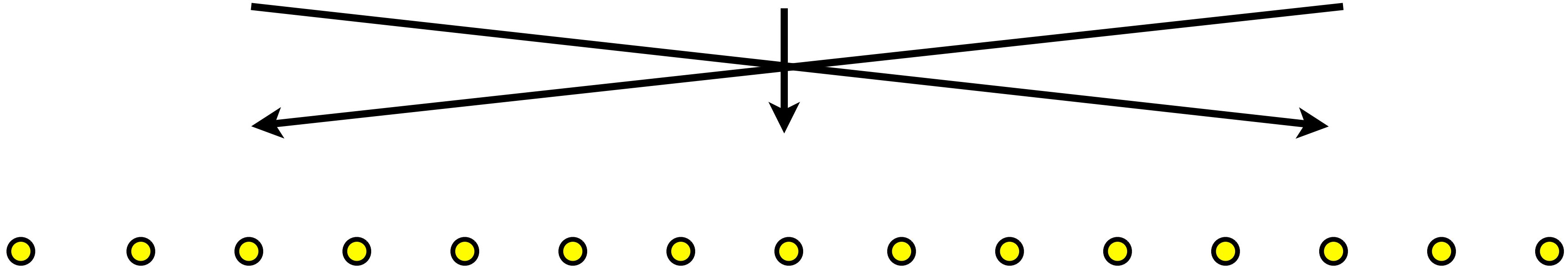
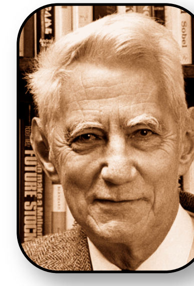


$$\text{Median} = f_A(1)\ f_B(f_A(1))\ f_C(f_B(f_A(1)))$$

Reduction to Selection



$0\ 0\ 0 \times (3 - f_A(1)) \times 5$

$1\ 0\ 0 \times (3 - f_B(1))$

$1\ 1\ f_C(1)$

$1\ 2\ f_C(2)$

$1\ 3\ f_C(3)$

$1\ 4\ 0 \times (f_B(1) - 1)$

$2\ 0\ 0 \times (3 - f_B(2))$

$2\ 1\ f_C(1)$

$2\ 2\ f_C(2)$

$2\ 3\ f_C(3)$

$2\ 4\ 0 \times (f_B(2) - 1)$

$3\ 0\ 0 \times (3 - f_B(3))$

$3\ 1\ f_C(1)$

$3\ 2\ f_C(2)$

$3\ 3\ f_C(3)$

$3\ 4\ 0 \times (f_B(3) - 1)$

$4\ 0\ 0 \times (f_A(1) - 1) \times 5$

VALUE

The Result

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- *Thm:* Finding an m^{δ}-approximate median in p passes requires $\Omega(m^{(1-\delta)/p}\, p^{-6})$ space.

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a) 1-pass, O(polylog m)-space, $\tilde{O}(m^{1/2})$-approx

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- *Bonus Thm:* For a stream in *random order*, a single pass, t-approx. requires $\Omega(m^{1/2} t^{-3/2})$ space.

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- *Bonus Thm:* For a stream in *random order*, a single pass, t-approx. requires $\Omega(m^{1/2} t^{-3/2})$ space.
- *Future Work:* Multi-pass ROM, trade-offs, *the brave new world of random-order streaming...*