Coming up

- Final projects:
  - User report are graded
  - All 1.0 presentations will be held on
  - Thursday, December 8, in class
  - 1.0 Release due data extended: Friday, December 9, 11:59PM
- Final exam:
  - Wednesday, December 14, 10:30 AM, in this room

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Today's plan

- Exam review
- Evaluations
- Reasoning about programs

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What'll be on the exam? (12/14, 10:30AM, here)

- Regular questions:
  - testing
  - debugging
  - working in groups
  - reasoning about programs
- high-level questions only:
  - software bias and formal verification of software
  - guest lectures on safety in software design

You may bring a single sheet (double side, 8.5" by 11" paper) of notes of your choosing.

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Exam: What kind of questions?

- True/False
- Multiple Choice
  (most are "choose all that apply")
- That's it. No other types of questions.

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Testing

- Know about different kinds of tests
  - unit, integration, regression, etc.
- Know about different kinds of coverage
  - statement, path, etc.
- Know what's hard about testing
  - GUI, usability, covering all behavior, etc.

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Debugging

- Know four kinds of defense against bugs
  - make impossible
  - don't introduce
  - make errors visible
  - last resort: debugging
- Representation (rep) invariants
- Assertions

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Working in groups

- What's hard?
  - corner cases
  - complete specification covers A LOT of behavior
  - unless a spec is concise, it's hard to understand
  - precision is hard: language is ambiguous
  - communication is important

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Reasoning about programs

- Ways to verify your code
  - testing, reasoning, proving
- Forward reasoning
- Backward reasoning
- Loop invariants
- Induction
- Practice some examples!

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Loop example

Find the weakest precondition

```
for (int x = 1; x <> y;) {
  if (y > x) {
    y = y / 2;
    x=2*x;
  }
}
```

// postcondition: x=8, y=8, and x and y are ints

you can also find the loop invariant and decrement function

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Evaluations

- We'll take 15 minutes to do evaluations
- They are anonymous and I don't see them until (long) after the grades are posted
- I actually use them to improve my teaching
- UMass uses them to decide if I am a good teacher and whether to let me keep teaching

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Bias in evaluations

- Ample scientific evidence that there are biases in evaluations.
- Women and minority faculty get statistically lower scores even when the teaching style is controlled to be exactly the same.
- Being aware is one of the best ways to combat the problem.

- MacNell et al. What's in a Name: Exposing Gender Bias in Student Ratings of Teaching. Innovative Higher Education, 2014, http://dx.doi.org/10.1007/s10755-014-9313-4
- Russ et al. Coming Out in the Classroom … An Occupational Hazard?: The Influence of Sexual Orientation on Teacher Credibility and Perceived Student Learning. Communication Education 51(3), 2002, 311–324. http://dx.doi.org/10.1080/03634520216516

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Evaluations

http://owl.umass.edu/partners/courseEvalSurvey/uma/

- If we get 80% participation by tomorrow:
  - Everyone gets 0.5 points of extra credit.

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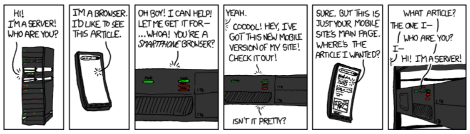
Reasoning about programs



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Ways to verify your code

- The hard way:
  - Make up some inputs
  - If it doesn't crash, ship it
  - When it fails in the field, attempt to debug
- The easier way:
  - Reason about possible behavior and desired outcomes
  - Construct simple tests that exercise that behavior
- Another way that can be easy
  - Prove that the system does what you want
    - Rep invariants are preserved
    - Implementation satisfies specification
  - Proof can be formal or informal (we will be informal)
  - Complementary to testing

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Reasoning about code

- Determine what facts are true during execution
  - x > 0
  - for all nodes n: n.next.previous == n
  - array a is sorted
  - x + y == z
  - if x != null, then x.a > x.b
- Applications:
  - Ensure code is correct (via reasoning or testing)
  - Understand why code is incorrect

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Forward reasoning

- You know what is true before running the code
  What is true after running the code?
- Given a precondition, what is the postcondition?
- Applications:
  Representation invariant holds before running code
  Does it still hold after running code?
- Example:
  // precondition: x is even
  x = x + 3;
  y = 2x;
  x = 5;
  // postcondition: ??

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Backward reasoning

- You know what you want to be true after running the code
  What must be true beforehand in order to ensure that?
- Given a postcondition, what is the corresponding precondition?
- Applications:
  (Re-)establish rep invariant at method exit: what's required?
  Reproduce a bug: what must the input have been?
- Example:
  // precondition: ??
  x = x + 3;
  y = 2x;
  x = 5;
  // postcondition: y > x
- How did you (informally) compute this?

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Forward vs. backward reasoning

- Forward reasoning is more intuitive for most people
  - Helps understand what will happen (simulates the code)
  - Introduces facts that may be irrelevant to goal
    Set of current facts may get large
  - Takes longer to realize that the task is hopeless
- Backward reasoning is usually more helpful
  - Helps you understand what should happen
  - Given a specific goal, indicates how to achieve it
  - Given an error, gives a test case that exposes it

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Forward reasoning example

```
assert x >= 0;
i = x;
    // x ≥ 0  &  i = x
z = 0;
    // x ≥ 0  &  i = x  &  z = 0
while (i != 0) {        ⇐ What property holds here?
  z = z + 1;
  i = i –1;             ⇐ What property holds here?
}
    // x ≥ 0  &  i = 0  &  z = x
assert x == z;
```

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Backward reasoning

Technique for backward reasoning:

- Compute the weakest precondition (wp)
- There is a wp rule for each statement in the programming language
- Weakest precondition yields strongest specification for the computation (analogous to function specifications)

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Assignment

```
// precondition: ??
x = e;
// postcondition: Q
```

Precondition: Q with all (free) occurrences of x replaced by e

- Example:

```
// assert:  ??
x = x + 1;
// assert x > 0

Precondition =  (x+1) > 0
```

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Method calls

```
// precondition: ??
x = foo();
// postcondition: Q
```

- If the method has no side effects: just like ordinary assignment
- If it has side effects: an assignment to every variable it modifies

Use the method specification to determine the new value

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If statements

```
// precondition:  ??
if (b) S1 else S2
// postcondition: Q
```

Essentially case analysis:

wp("if (b) S1 else S2", Q) =

$(\quad b \Rightarrow wp(\text{"S1"}, Q)$

$\wedge \neg b \Rightarrow wp(\text{"S2"}, Q)\ )$

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If: an example

```
// precondition: ??
if (x == 0) {
   x = x + 1;
} else {
   x = (x/x);
}
// postcondition:  x ≥ 0
```

Precondition:

wp("if (x==0) {x = x+1} else {x = x/x}", x ≥ 0) =

= (x = 0 ⇒ wp("x = x+1", x ≥ 0)

  & x ≠ 0 ⇒ wp("x = x/x", x ≥ 0))

= (x = 0 ⇒ x + 1 ≥ 0) & (x ≠ 0 ⇒ x/x ≥ 0)

= 1 ≥ 0 & 1 ≥ 0

= true

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Reasoning About Loops

- A loop represents an unknown number of paths
  - Case analysis is problematic
  - Recursion presents the same issue
- Cannot enumerate all paths
  - That is what makes testing and reasoning hard

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Loops: values and termination

```
// assert x ≥ 0 & y = 0
while (x != y) {
    y = y + 1;
}
// assert x = y
```

1) Pre-assertion guarantees that $x \geq y$
2) Every time through loop
   $x \geq y$ holds and, if body is entered, $x > y$
   y is incremented by 1
   x is unchanged
   Therefore, y is closer to x (but $x \geq y$ still holds)
3) Since there are only a finite number of integers between x and y, y will eventually equal x
4) Execution exits the loop as soon as x = y

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Understanding loops by induction

- We just made an inductive argument
  Inducting over the number of iterations
- Computation induction
  Show that conjecture holds if zero iterations
  Assume it holds after n iterations and show it holds after n+1
- There are two things to prove:
  Some property is preserved (known as "partial correctness")
    loop invariant is preserved by each iteration
  The loop completes (known as "termination")
    The "decrementing function" is reduced by each iteration

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Loop invariant for the example

```
// assert x ≥ 0 & y = 0
while (x != y) {
    y = y + 1;
}
// assert x = y
```

- So, what is a suitable invariant?
- What makes the loop work?
  LI = $x \geq y$

  1) $x \geq 0$ & $y = 0 \Rightarrow$ LI
  2) LI & $x \neq y$ {`y = y+1;`} LI
  3) (LI & $\neg(x \neq y)$) $\Rightarrow$ $x = y$

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Is anything missing?

```
// assert x ≥ 0 & y = 0
while (x != y) {
    y = y + 1;
}
// assert x = y
```

Does the loop terminate?

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Decrementing Function

- Decrementing function D(X)
  - Maps state (program variables) to some well-ordered set
  - This greatly simplifies reasoning about termination
- Consider: `while (b) S;`
- We seek D(X), where X is the state, such that
  1. An execution of the loop reduces the function's value:
     LI & b {**S**} $D(X_{post}) < D(X_{pre})$
  2. If the function's value is minimal, the loop terminates:
     (LI & D(X) = minVal) $\Rightarrow \neg$b

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Proving Termination

```
// assert x ≥ 0 & y = 0
// Loop invariant: x ≥ y
// Loop decrements: (x-y)
while (x != y) {
    y = y + 1;
}
// assert x = y
```

- Is "x-y" a good decrementing function?

1. Does the loop reduce the decrementing function's value?
   // assert (y ⍰ x); let d_{pre} = (x – y)
   y = y + 1;
   // assert (x_{post} – y_{post}) < d_{pre}
2. If the function has minimum value, does the loop exit?
   (x ⍰ y & x – y = 0) ⍰ (x = y)

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Choosing Loop Invariant

- For straight-line code, the wp (weakest precondition) function gives us the appropriate property
- For loops, you have to guess:
  - The loop invariant
  - The decrementing function
- Then, use reasoning techniques to prove the goal property
- If the proof doesn't work:
  - Maybe you chose a bad invariant or decrementing function
    - Choose another and try again
  - Maybe the loop is incorrect
    - Fix the code
- Automatically choosing loop invariants is a research topic

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In practice

I don't routinely write loop invariants

I do write them when I am unsure about a loop and when I have evidence that a loop is not working

- Add invariant and decrementing function if missing
- Write code to check them
- Understand why the code doesn't work
- Reason to ensure that no similar bugs remain

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More on Induction

- Induction is a very powerful tool

$$2^n = 1 + \sum_{k=1}^{n} 2^{k-1}$$

Proof by induction: Base Case

For n=1, $1 + \sum_{k=1}^{1} 2^{k-1} = 1 + 2^0 = 1 + 1 = 2 = 2^1$

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Inductive Step

Assume $2^m = 1 + \sum_{k=1}^{m} 2^{k-1}$ and show that $2^{m+1} = 1 + \sum_{k=1}^{m+1} 2^{k-1}$

$$2^{m+1} = 1 + \sum_{k=1}^{m+1} 2^{k-1} = 1 + \sum_{k=1}^{m} 2^{k-1} + 2^m = 2^m + 2^m = 2 \times 2^m = 2^{m+1}$$

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Is Induction Too Powerful?

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