9/13/16 LM Part 1
CS 585
UMass Amherst

Slides: see J+M 3rd ed webpage

Probabilistic Language Models

- Today's goal: assign a probability to a sentence

Why?

- Machine Translation:
  - P(**high** winds tonite) > P(**large** winds tonite)
- Spell Correction
  - The office is about fifteen **minuets** from my house
    - P(about fifteen **minutes** from) > P(about fifteen **minuets** from)
- Speech Recognition
  - P(I saw a van) >> P(eyes awe of an)
- + Summarization, question-answering, etc., etc.!!

+ Predictive Text Entry

Probabilistic Language Modeling

- Goal: compute the probability of a sentence or sequence of words:

$$P(W) = P(w_1,w_2,w_3,w_4,w_5 \dots w_n)$$

- Related task: probability of an upcoming word:

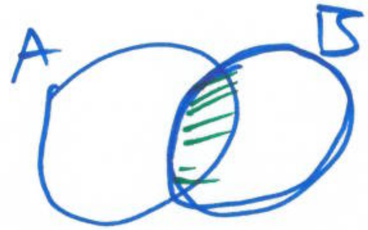
$P(w_5 | w_1,w_2,w_3,w_4)$ $\quad P(w_5 = \text{UFO} \mid w_1 = \text{I},\ w_2 = \text{saw},\ w_3 = \text{the},\ w_4 = \text{red})$

- A model that computes either of these:

$P(W)$ or $P(w_n | w_1,w_2 \dots w_{n-1})$ is called a **language model**.

- Better: **the grammar** But **language model** or **LM** is standard

Def. Cond Prob: $P(A|B) = \frac{P(A \cap B)}{P(B)}$



<u>Chain Rule</u>: $P(AB) = P(A|B)\,P(B)$

$P(A,B,C) = P(A|BC) \cdot P(B|C) \cdot P(C)$

How to compute P(W)

- How to compute this joint probability:
  - P(its, water, is, so, transparent, that)
- Intuition: let's rely on the Chain Rule of Probability

$$P(A,B,C) = P(C|AB)\,P(AB)$$
$$= P(C|AB)\,P(B|A)\,P(A)$$

The Chain Rule applied to compute joint probability of words in sentence

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i \mid w_1 w_2 \ldots w_{i-1})$$

P("its water is so transparent") =

P(its) × P(water|its) × P(is|its water)

× P(so|its water is) × P(transparent|its water is so)

$P(w_2 = water \mid w_1 = its)$

How to estimate these probabilities

- Could we just count and divide?

$$P(\text{the} \mid \text{its water is so transparent that}) = \frac{Count(\text{its water is so transparent that the})}{Count(\text{its water is so transparent that})}$$

- No! Too many possible sentences!
- We'll never see enough data for estimating these

Markov Assumption

Andrei Markov

- Simplifying assumption:

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{that})$$

$$\text{estimate} := \frac{\text{count}(\cancel{\text{the, that}}\ \text{that the})}{\text{count}(\cancel{\text{the}}\ \text{that})}$$

- Or maybe

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{transparent that})$$

Markov Assumption

$$P(w_i \mid w_1, w_2 \dots w_{i-1}) \approx P(w_i \mid \underbrace{w_{i-k}, w_{i-k+1}, \dots w_{i-1}}_{\text{length } k \text{ history}})$$

kth-order Markov Assump.

$$w\ \ w\ \ w\ \ w\ \ w\ \ w\ \ \underbrace{w\ \ w}_{\substack{k=2\\ \text{history}}}\ \ \underbrace{w_i}$$

$$P(w_1 \dots w_n) = \prod_i P(w_i \mid w_{i-k}, \dots w_{i-1})$$

Bigram model

1st order Markov model

- Condition on the previous word:

$$P(w_i \mid w_1 w_2 \ldots w_{i-1}) \approx P(w_i \mid w_{i-1})$$

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texaco, rose, one, in, this, issue, is, pursuing, growth, in,
a, boiler, house, said, mr., gurria, mexico, 's, motion,
control, proposal, without, permission, from, five, hundred,
fifty, five, yen

outside, new, car, parking, lot, of, the, agreement, reached

this, would, be, a, record, november
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Simplest case: Unigram model

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i)$$

Some automatically generated sentences from a unigram model

```
fifth, an, of, futures, the, an, incorporated, a,
a, the, inflation, most, dollars, quarter, in, is,
mass

thrift, did, eighty, said, hard, 'm, july, bullish

that, or, limited, the
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N-gram models

- We can extend to trigrams, 4-grams, 5-grams
- In general this is an insufficient model of language
  - because language has **long-distance dependencies**:

  "The computer which I had just put into the machine room on the fifth floor crashed."

- But we can often get away with N-gram models

Estimating bigram probabilities

- The Maximum Likelihood Estimate

$$P(w_i \mid w_{i-1}) = \frac{count(w_{i-1}, w_i)}{count(w_{i-1})}$$

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

An example

start/end markers

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s> I am Sam </s>
<s> Sam I am </s>
<s> I do not like green eggs and ham </s>

$P(I \mid \text{<s>}) = \frac{2}{3} = 0.66$

$P(\text{Sam} \mid \text{<s>}) = \frac{1}{3}$

$P(\text{do} \mid \text{<s>}) = 0$

$P(\text{not} \mid \text{<s>}) = 0$

$\vdots \qquad \vdots$

More examples: Berkeley Restaurant Project sentences

- can you tell me about any good cantonese restaurants close by
- mid priced thai food is what i'm looking for
- tell me about chez panisse
- can you give me a listing of the kinds of food that are available
- i'm looking for a good place to eat breakfast
- when is caffe venezia open during the day

Raw bigram counts

- Out of 9222 sentences

c(i, to)

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Raw bigram probabilities

- Normalize by unigrams:

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

- Result:

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Size = $|V|^2$ N tokens in training set

Bigram estimates of sentence probabilities

P(<s> I want english food </s>) =

P(I|<s>)

× P(want|I)

× P(english|want)

× P(food|english)

× P(</s>|food)

= .000031

What kinds of knowledge?

- P(english|want) = .0011
- P(chinese|want) = .0065
- P(to|want) = .66
- P(eat | to) = .28
- P(food | to) = 0
- P(want | spend) = 0
- P (i | <s>) = .25

Practical Issues

- We do everything in log space
  - Avoid underflow
  - (also adding is faster than multiplying)

$$\log(p_1 \times p_2 \times p_3 \times p_4) = \log p_1 + \log p_2 + \log p_3 + \log p_4$$