Handout 2/22/17 (UMass CS 690N)
HMM example from the Jurafsky and Martin textbook (Jason Eisner's ice cream example)

Model

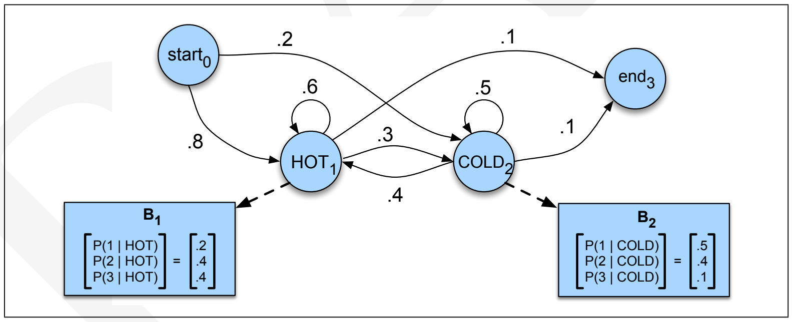


Figure 7.3 A hidden Markov model for relating numbers of ice creams eaten by Jason (the observations) to the weather (H or C, the hidden variables).

Forward-Backward

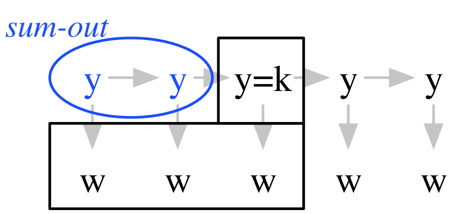
Declaratively:

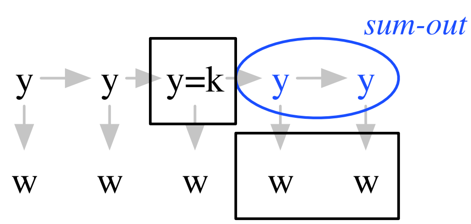
Forward probs

$$\alpha_t[k] = \sum_{y_1 \dots y_{t-1}} P(y_t = k, w_1..w_t, y_1..y_{t-1})$$

Backward probs

$$\beta_t[k] = \sum_{y_{t+1} \dots y_n} P(y_t = k, w_{t+1}..w_n, y_{t+1}..y_n)$$





Forward Algo.: for each *t=1..N,* for each *k*,

$$\alpha_t[k] := \sum_{j=1..K} \left(\alpha_{t-1}[j]\ P_{trans}(k \mid j)\ P_{emit}(w_t \mid k) \right)$$

Backward Algo.: for each *t=N..1*, for each *j*,

$$\beta_t[j] := \sum_{k=1..K} \left(\beta_{t+1}[j]\ P_{trans}(k \mid j)\ P_{emit}(w_{t+1} \mid k) \right)$$

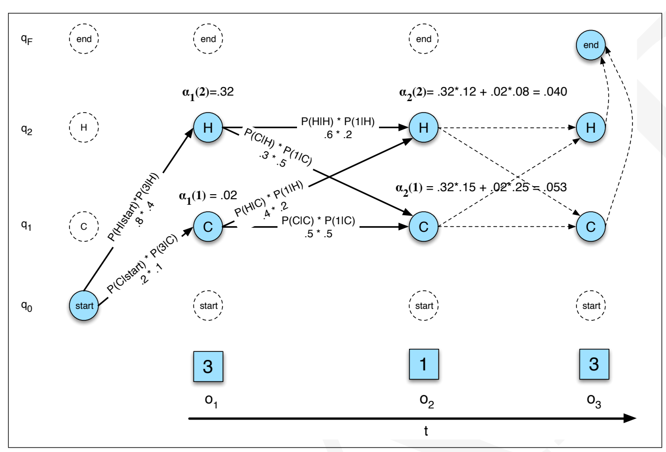
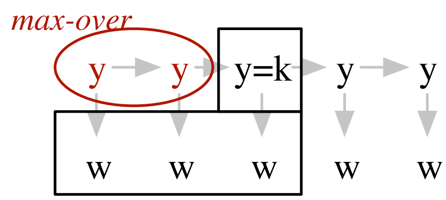


Figure 7.7 The forward trellis for computing the total observation likelihood for the ice-cream events *3 1 3*. Hidden states are in circles, observations in squares. White (unfilled) circles indicate illegal transitions. The figure shows the computation of $\alpha_t(j)$ for two states at two time steps. The computation in each cell follows Eq. 7.14: $\alpha_t(j) = \sum_{i=1}^{N} \alpha_{t-1}(i) a_{ij} b_j(o_t)$. The resulting probability expressed in each cell is Eq. 7.13: $\alpha_t(j) = P(o_1, o_2 \dots o_t, q_t = j|\lambda)$.

Viterbi algorithm (for HMMs)

Declaratively:

$$V_t[k] = \max_{y_1 \ldots y_{t-1}} P(y_t = k, \quad y_1..y_{t-1}, w_1..w_t)$$



Algorithm, for each *t=1..N,*

$$V_t[k] := \max_{j=1..K} \left(V_{t-1}[j] \; P_{trans}(k \mid j) \; P_{emit}(w_t \mid k) \right)$$

$$B_t[k] := \arg\max_{j=1..K} \left(\ldots \right)$$

For solution: choose best tag at last position.
Trace backpointers to find best tag at second-to-last, e tc.

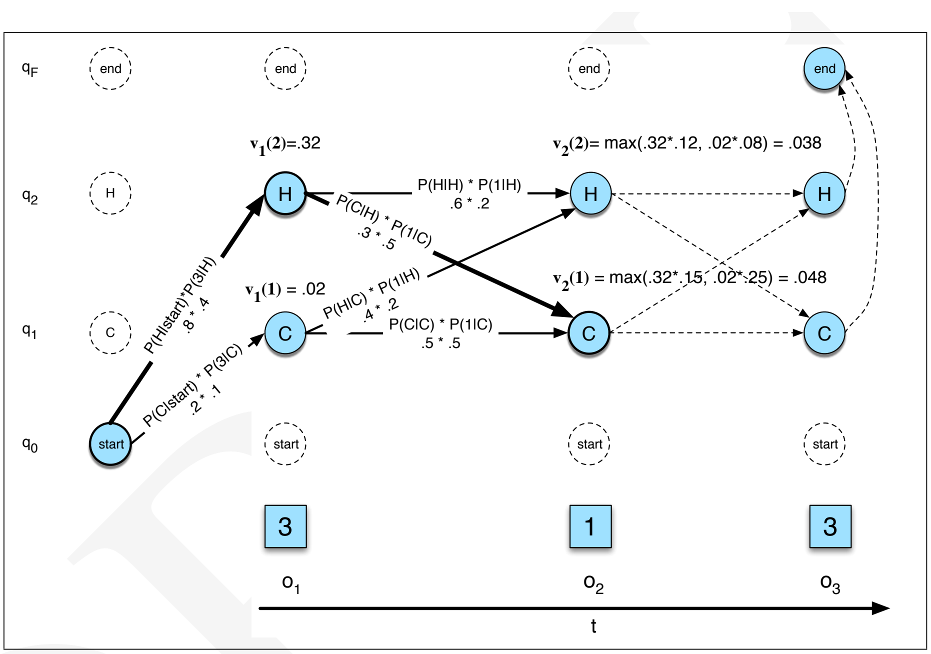


Figure 7.10 The Viterbi trellis for computing the best path through the hidden state space for the ice-cream eating events *3 1 3*. Hidden states are in circles, observations in squares. White (unfilled) circles indicate illegal transitions. The figure shows the computation of $v_t(j)$ for two states at two time steps. The computation in each cell follows Eq. 7.19: $v_t(j) = \max_{1 \leq i \leq N-1} v_{t-1}(i)\, a_{ij}\, b_j(o_t)$. The resulting probability expressed in each cell is Eq. 7.18: $v_t(j) = P(q_0, q_1, \ldots, q_{t-1}, o_1, o_2, \ldots, o_t, q_t = j|\lambda)$.